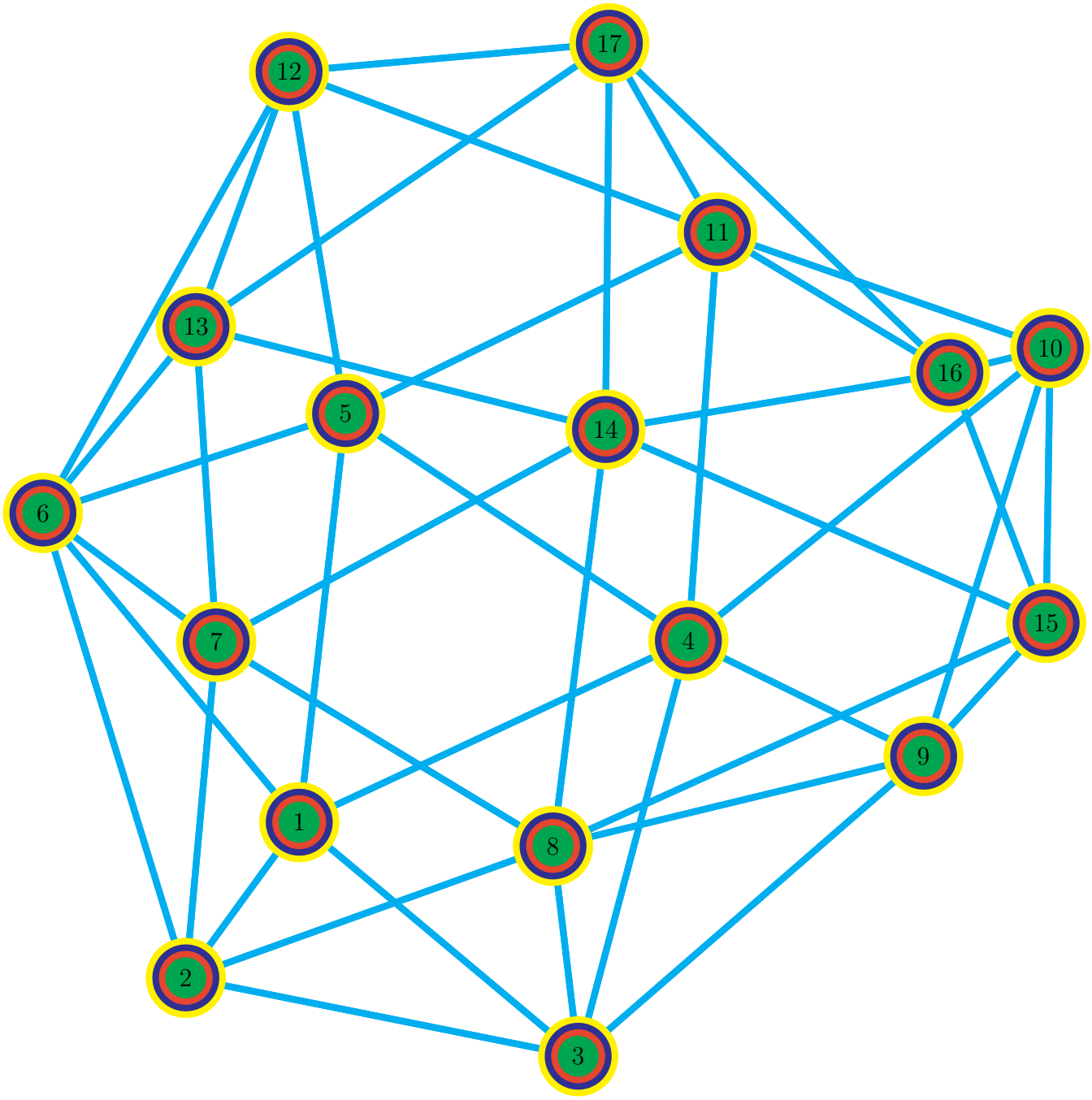

Figure 1: Instructions Begin:

we begin

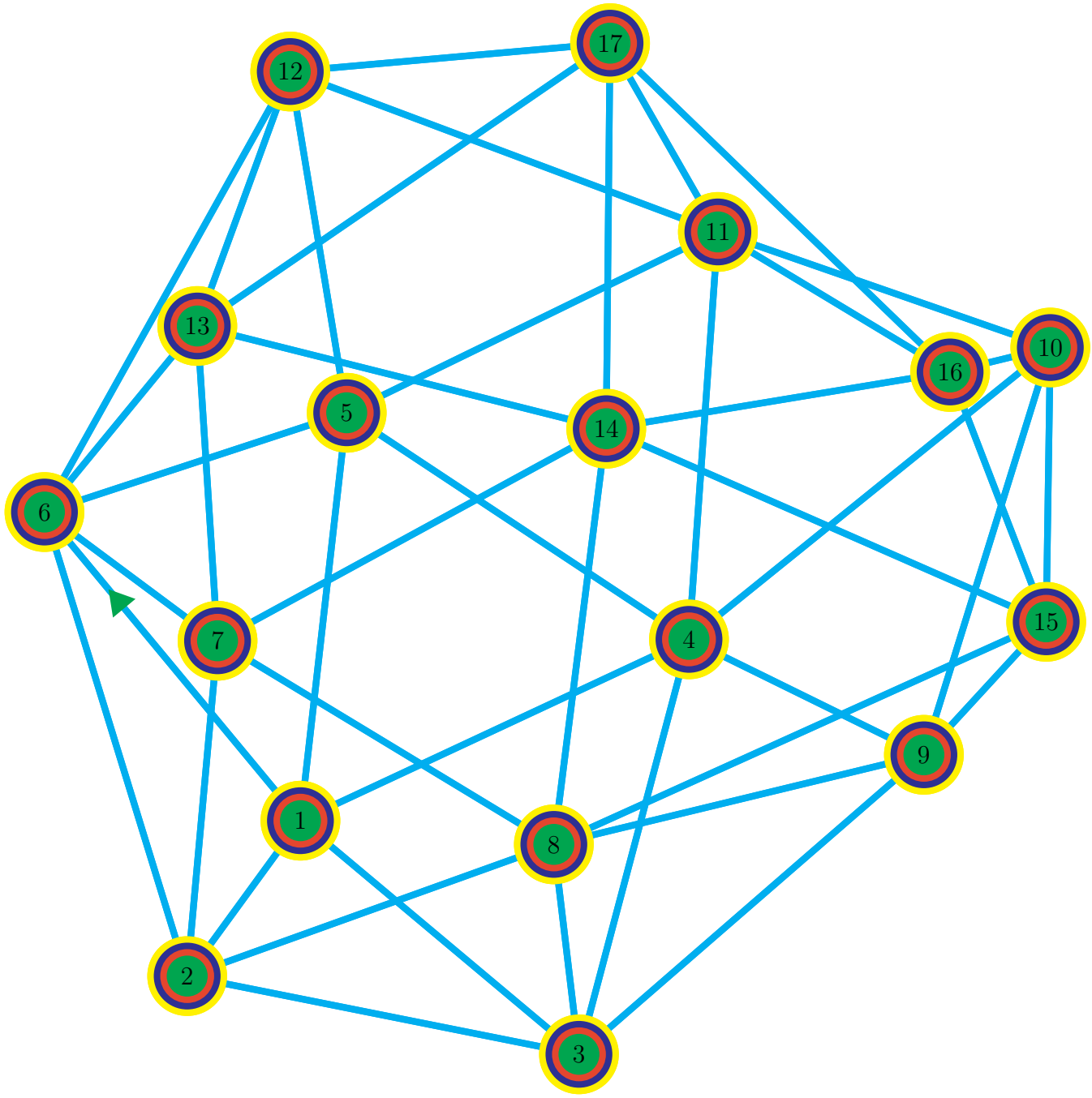

Figure 2: Instructions: 1: place edge 1→6 Green ArrowR

When the above instruction is executed by the computer a triangular green arrowhead is placed on edge 1→6. A triangular green arrowhead is used to indicate deletion of the green color on the vertex pointed to. (You can think of the green color on a vertex as a green checker.) Define triangle arrowheads as deletion arrows. Later we'll define insertion arrows.

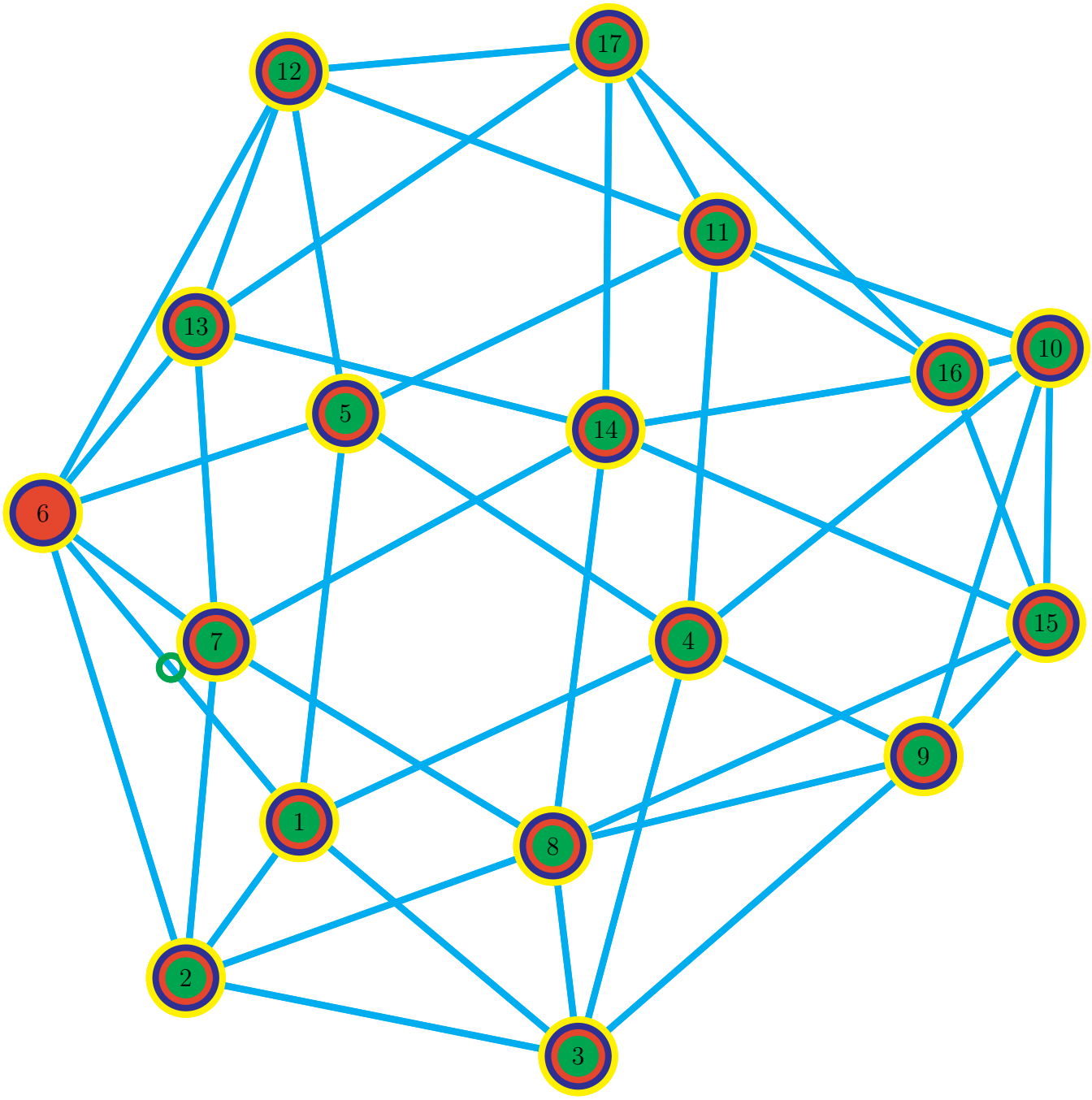

Figure 3: Instructions: 4: color edge 1→6 Green 5: uncolor vertex 6 Green

A circle is placed on edge 1→6 indicating that this edge is green colored. The green checker has been remove from vertex 6. Also the edge is conflict free wtr the color green ie there is only one green checker on vertices of this edge.

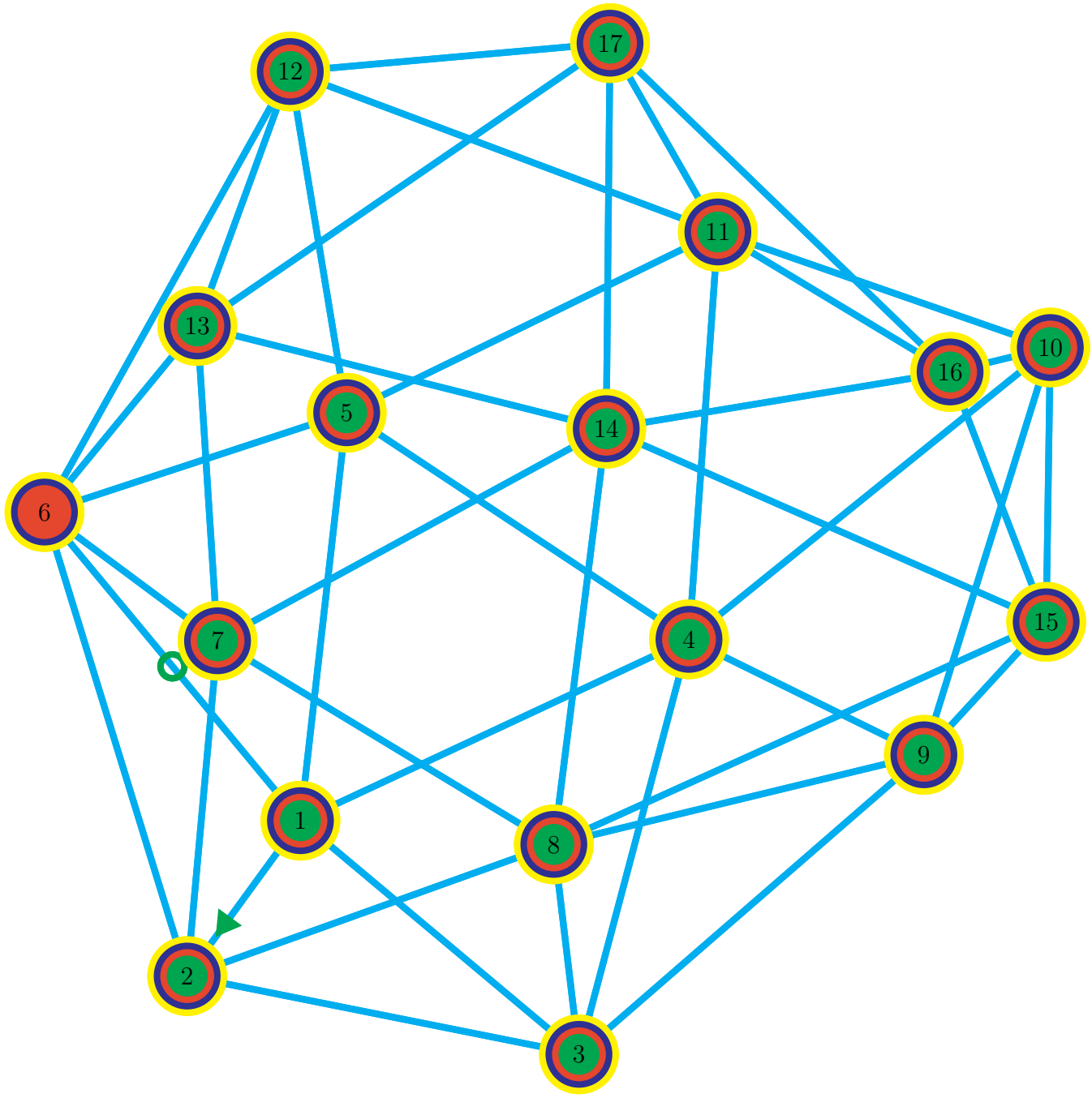

Figure 4: Instructions: 7: place edge 1-2 Green ArrowR

Continuing in this same manner we obtain a sequence of graphs with the following properties.
1: If a circle of a given color is on an edge there cannot be two checkers of the same color on vertices of that edge. Lets call this property A.
2: If a circle of a given color is on an edge there must be one and only one checker of that color on verices of that edge. Lets call this property B.

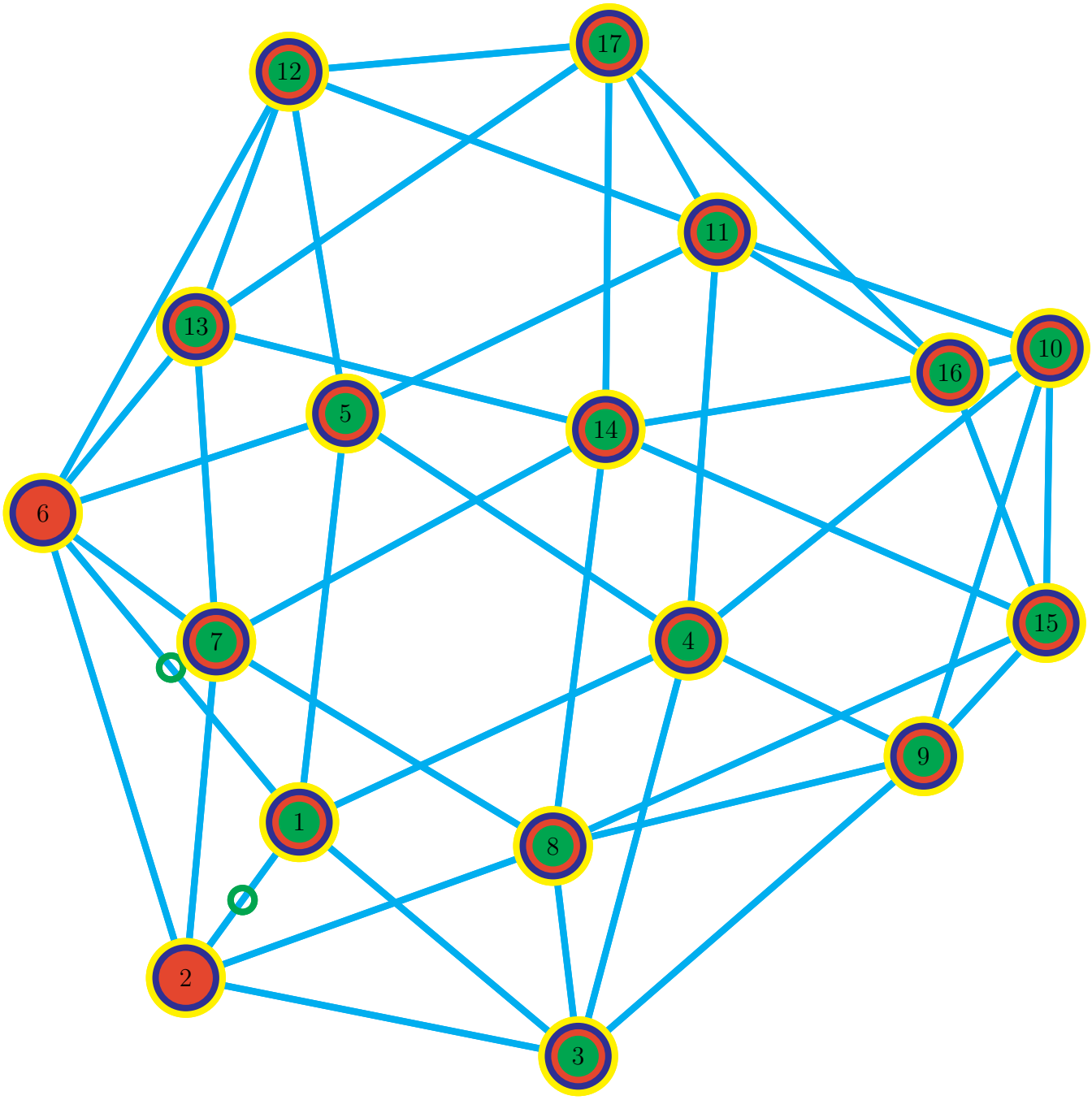

Figure 5: Instructions: 10: color edge 1→2 Green 11: uncolor vertex 2 Green

The green checker has been removed from vertex 2 and a green circle has been placed on edge 1-2 and properties A and B are are satisfied as they will be after each major iteration of the algorithm. I will have something further to say at instruction 244.

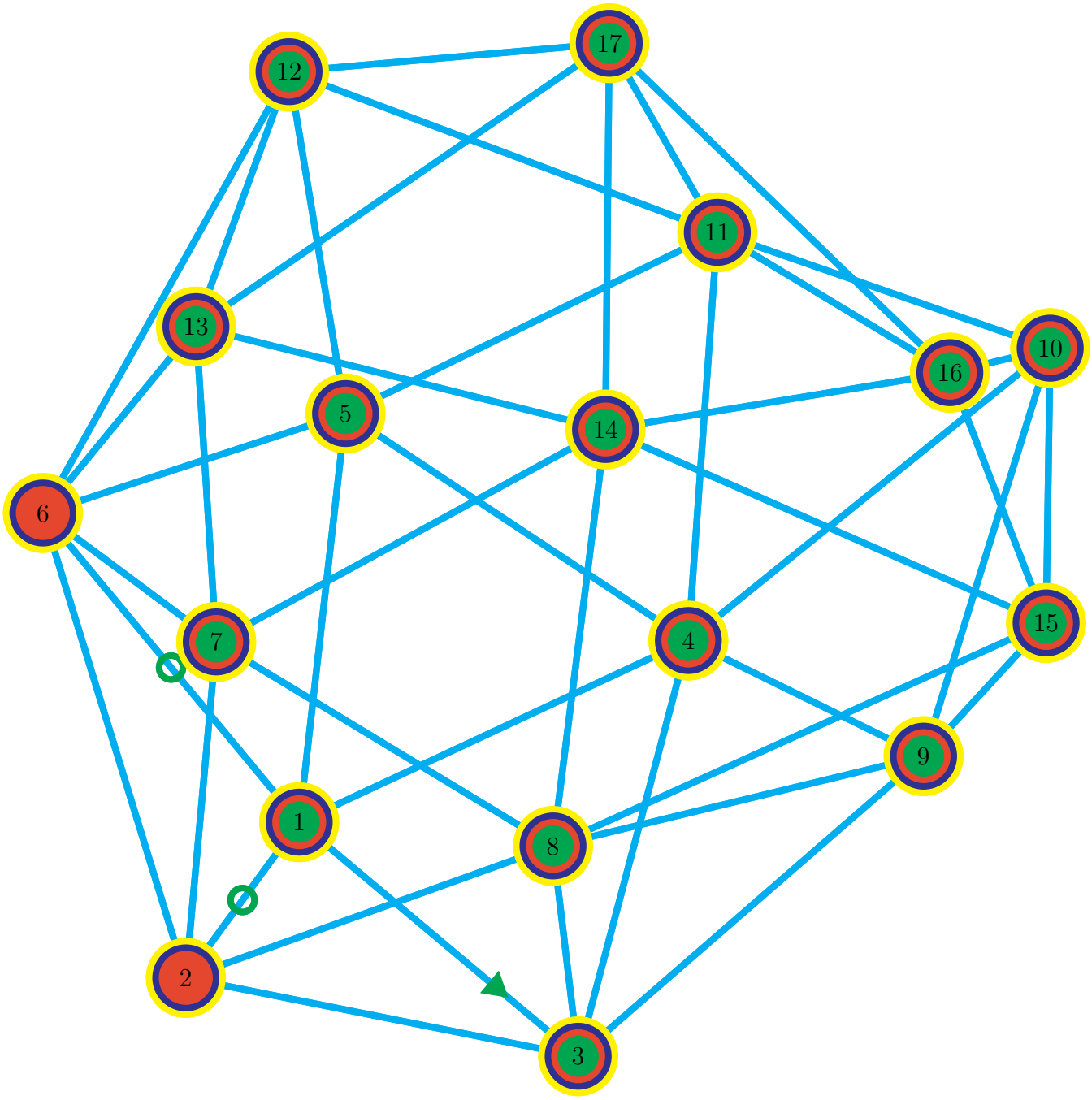

Figure 6: Instructions: 13: place edge 1→3 Green ArrowR

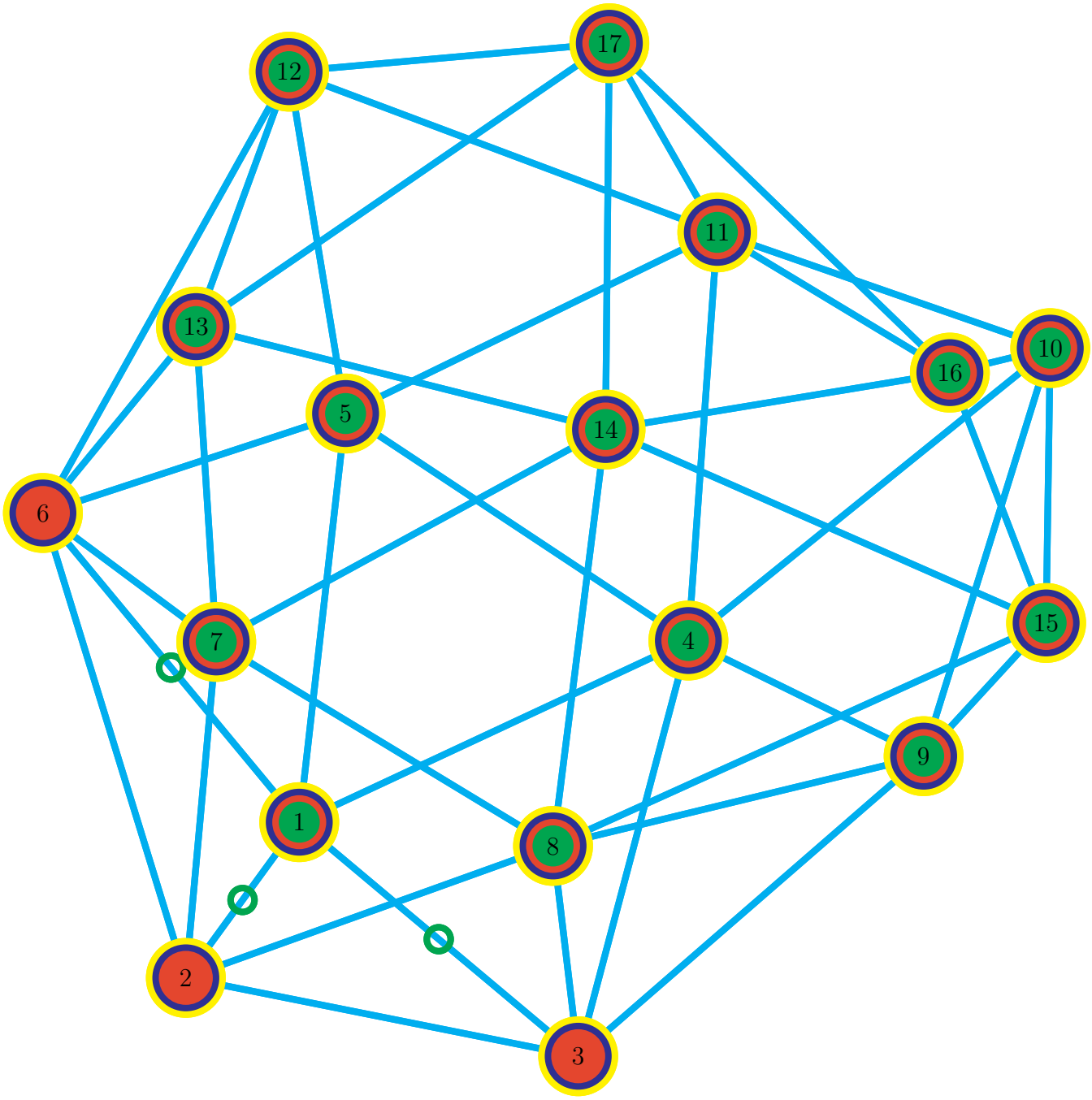

Figure 7: Instructions: 16: color edge 1→3 Green 17: uncolor vertex 3 Green

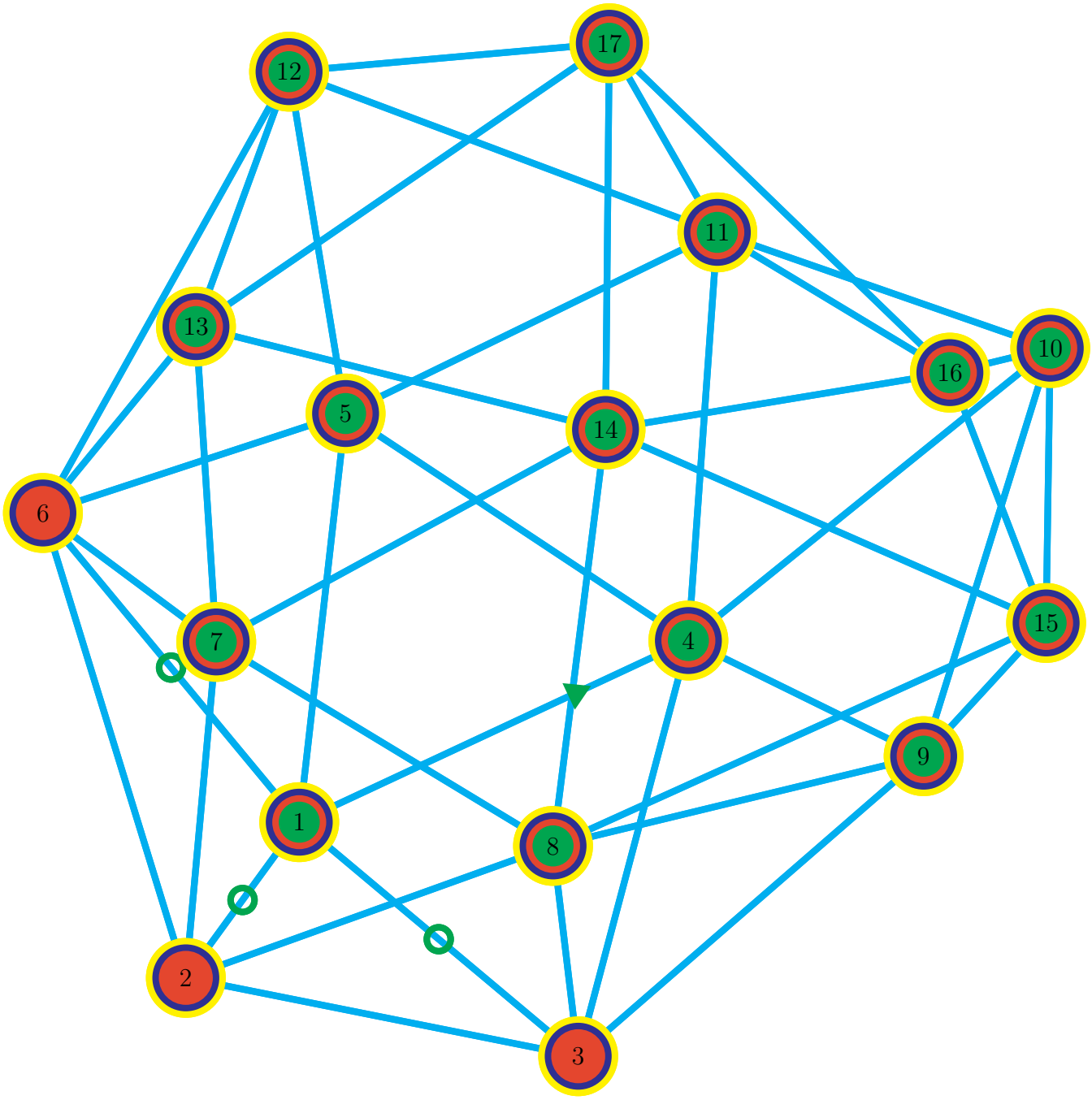

Figure 8: Instructions: 19: place edge 1→4 Green ArrowR

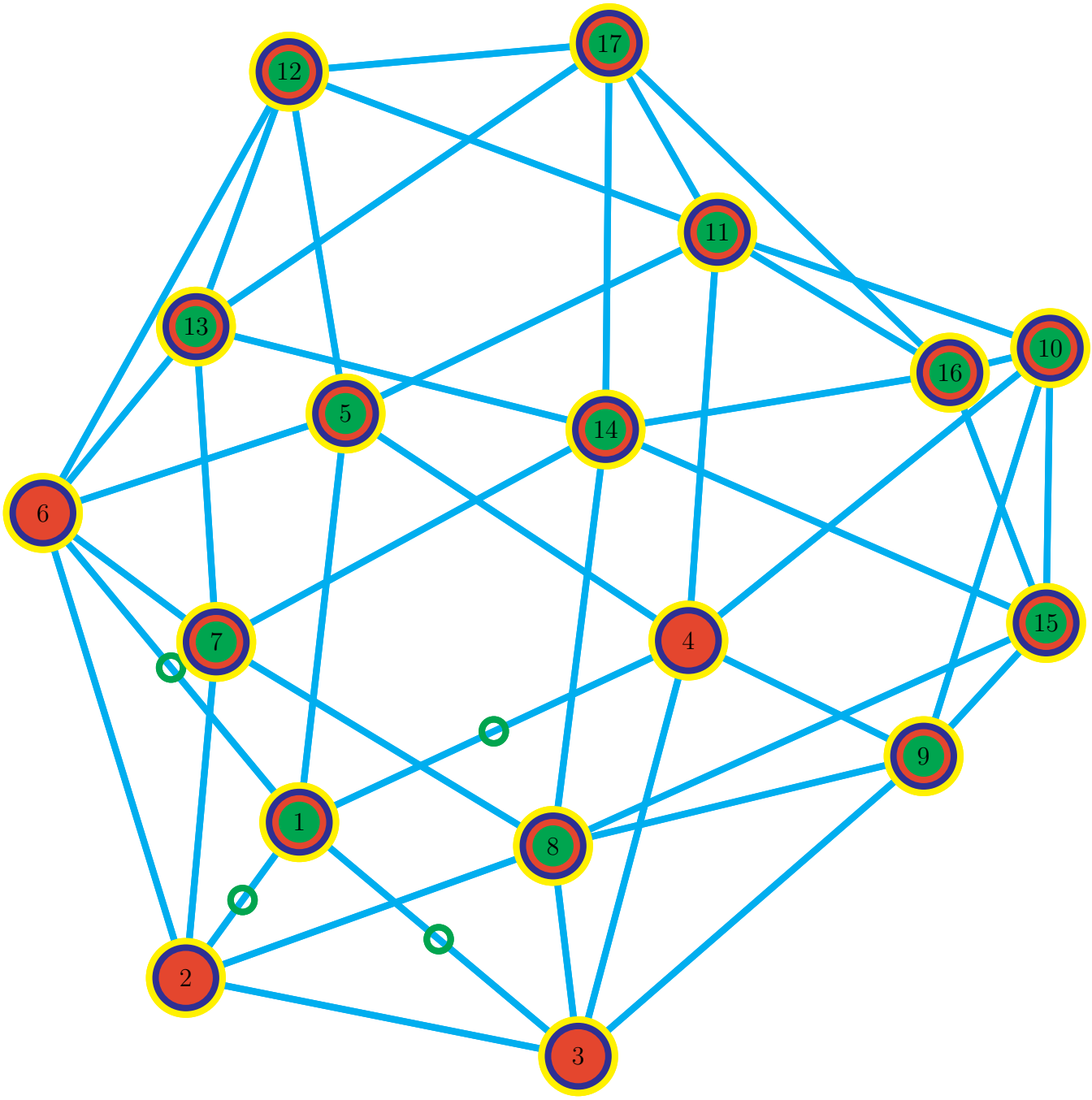

Figure 9: Instructions: 22: color edge 1→4 Green 23: uncolor vertex 4 Green

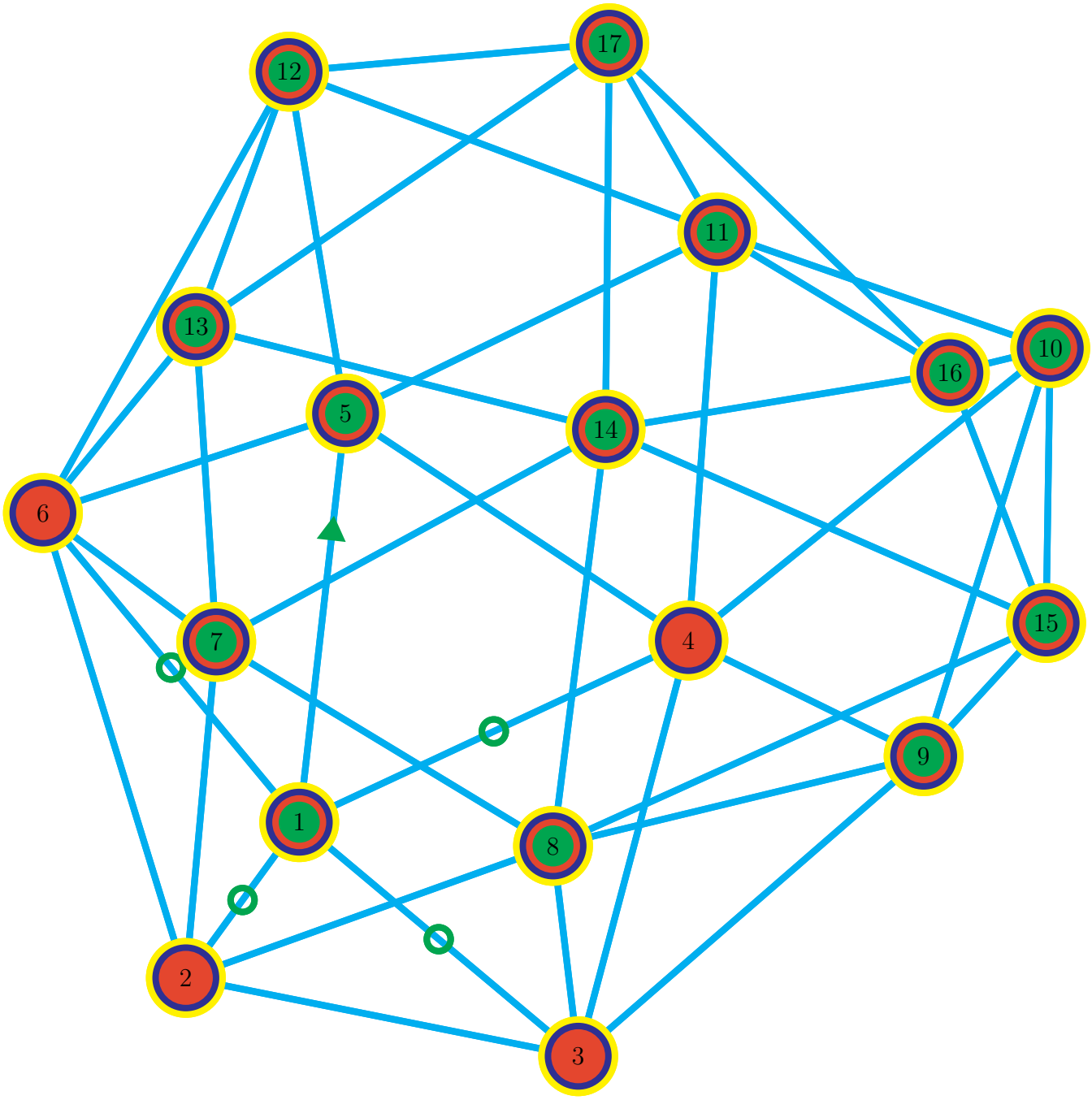

Figure 10: Instructions: 25: place edge 1→5 Green ArrowR

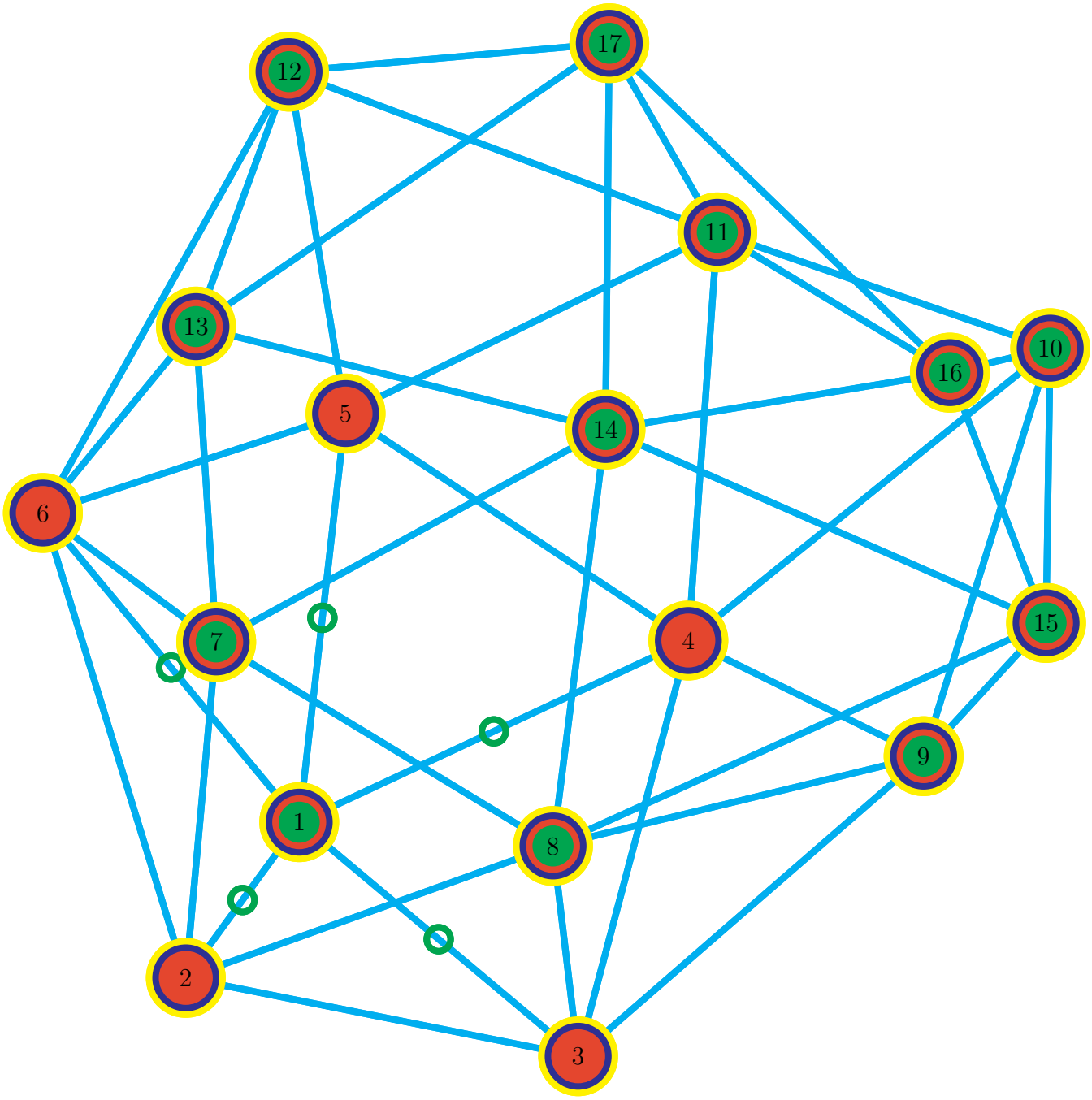

Figure 11: Instructions: 28: color edge 1→5 Green 29: uncolor vertex 5 Green

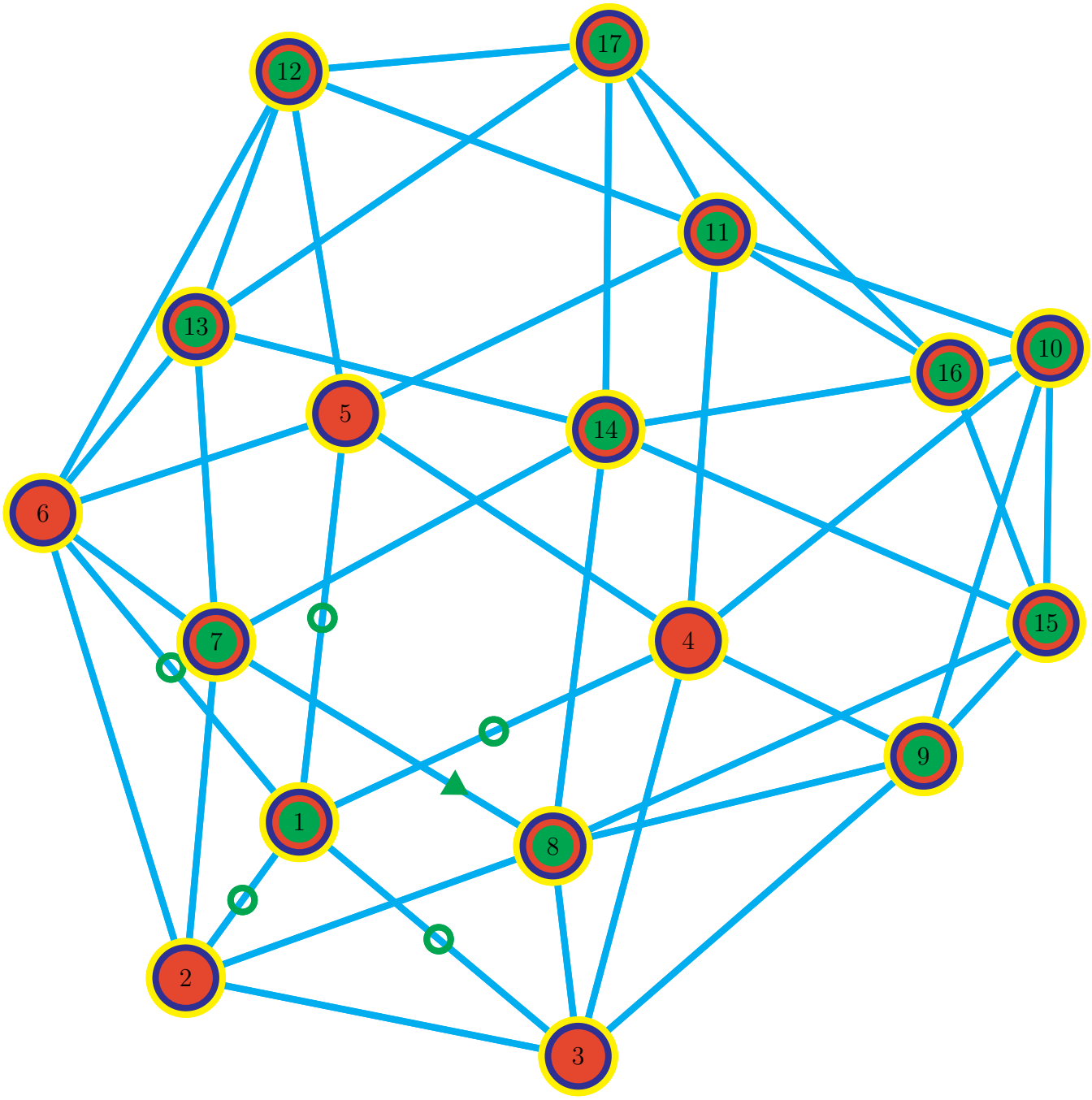

Figure 12: Instructions: 31: place edge 7→8 Green ArrowR

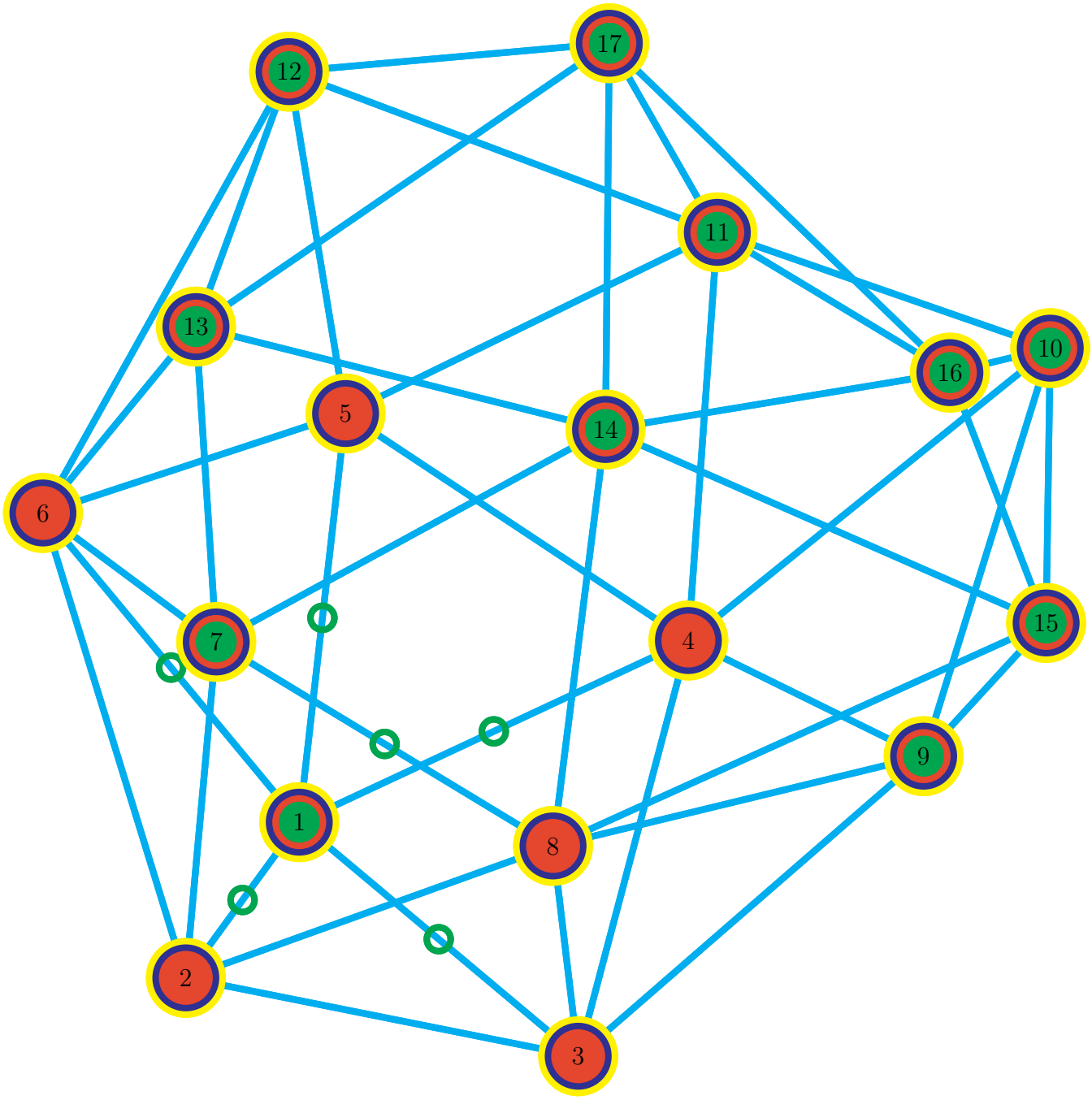

Figure 13: Instructions: 34: color edge 7→8 Green 35: uncolor vertex 8 Green

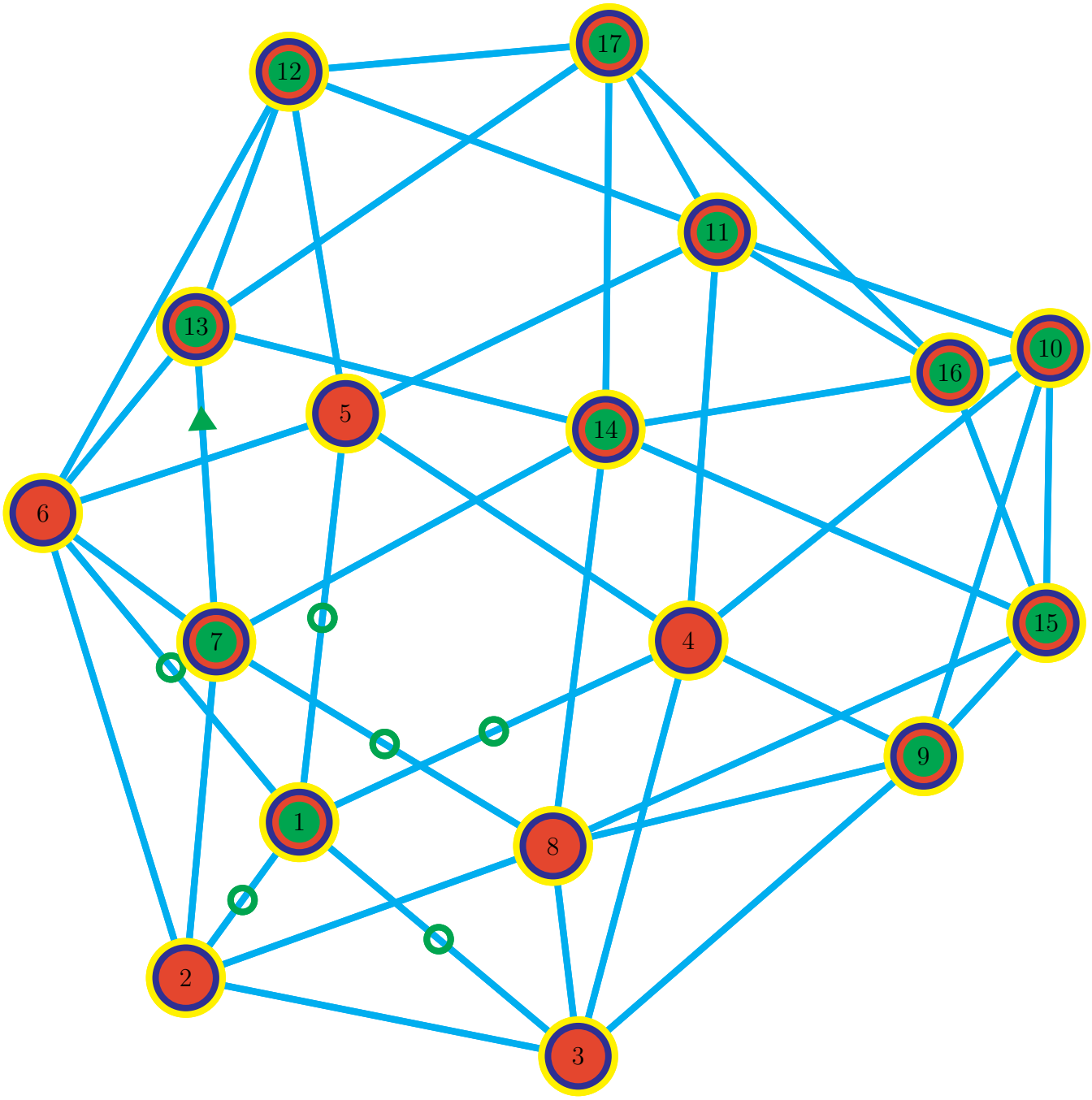

Figure 14: Instructions: 37: place edge 7→13 Green ArrowR

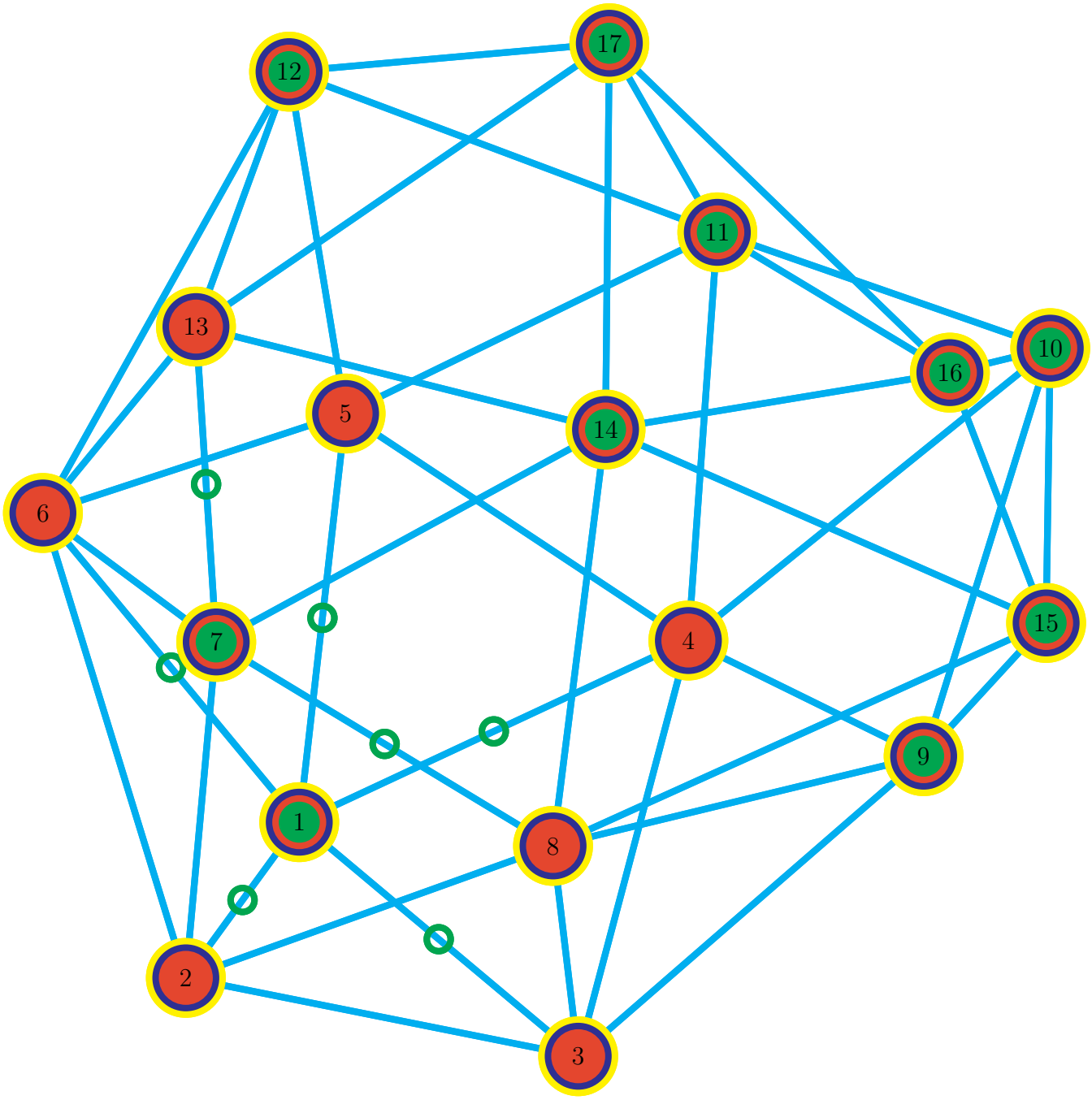

Figure 15: Instructions: 40: color edge 7→13 Green 41: uncolor vertex 13 Green

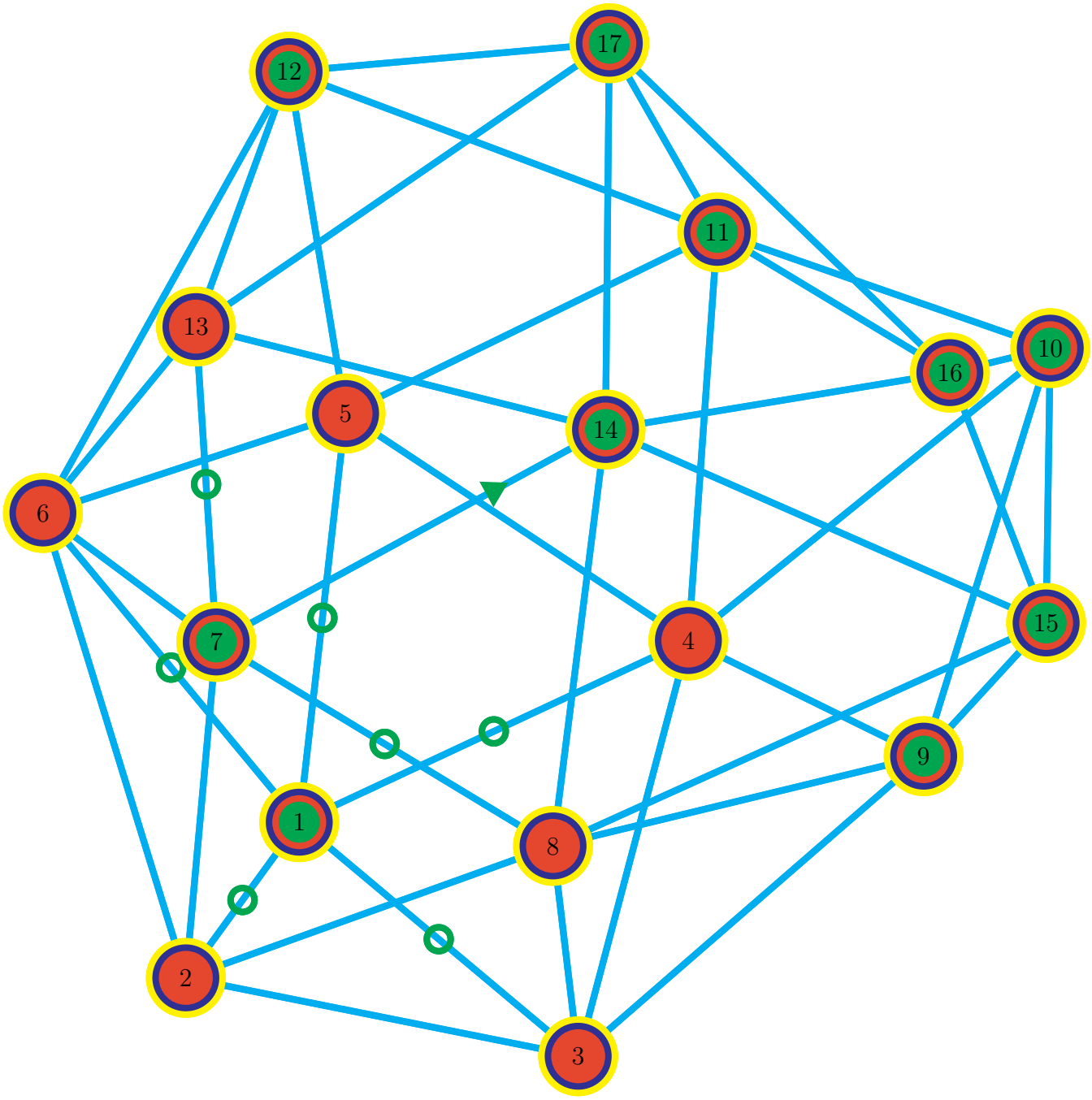

Figure 16: Instructions: 43: place edge 7→14 Green ArrowR

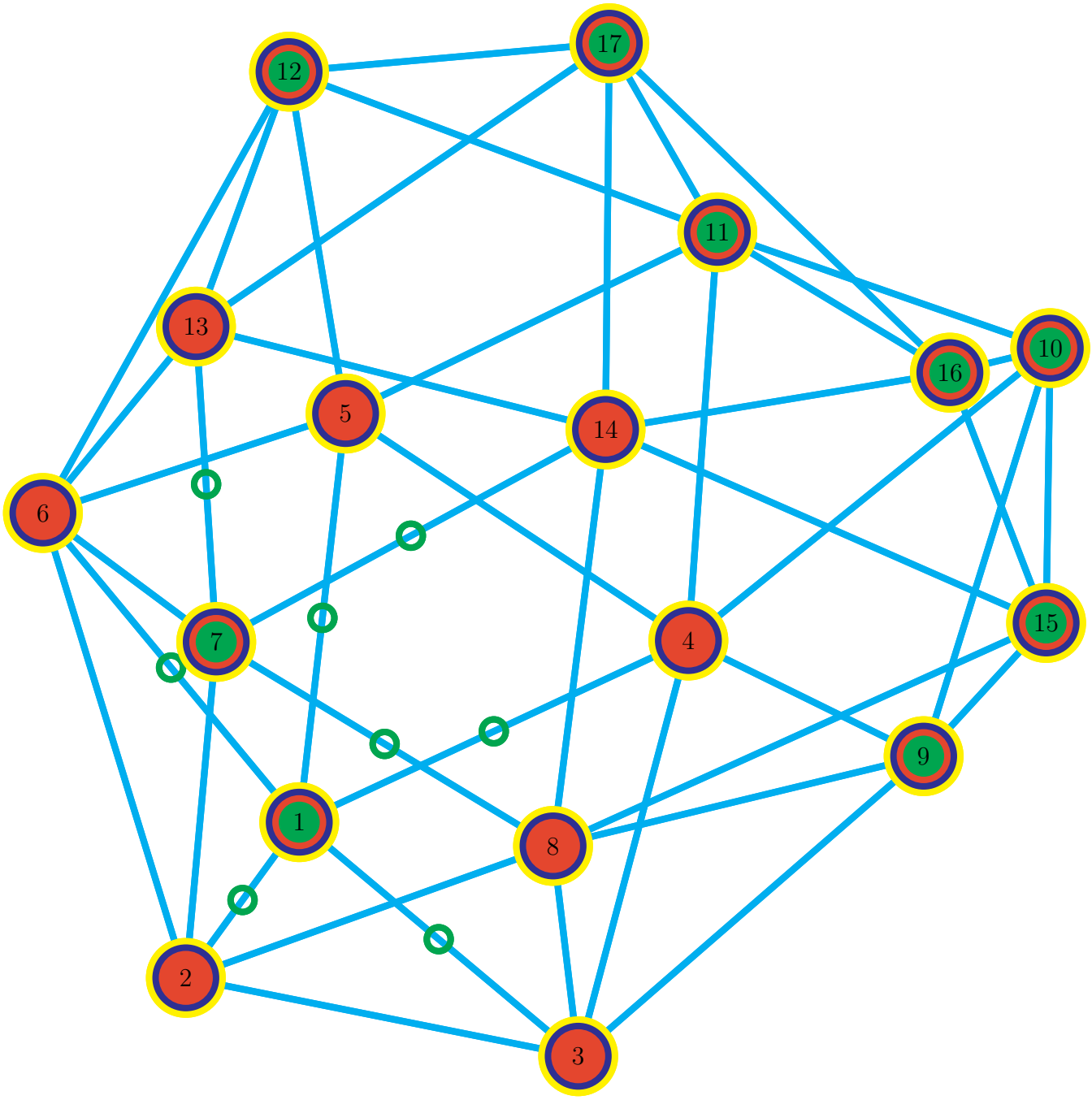

Figure 17: Instructions: 46: color edge 7→14 Green 47: uncolor vertex 14 Green

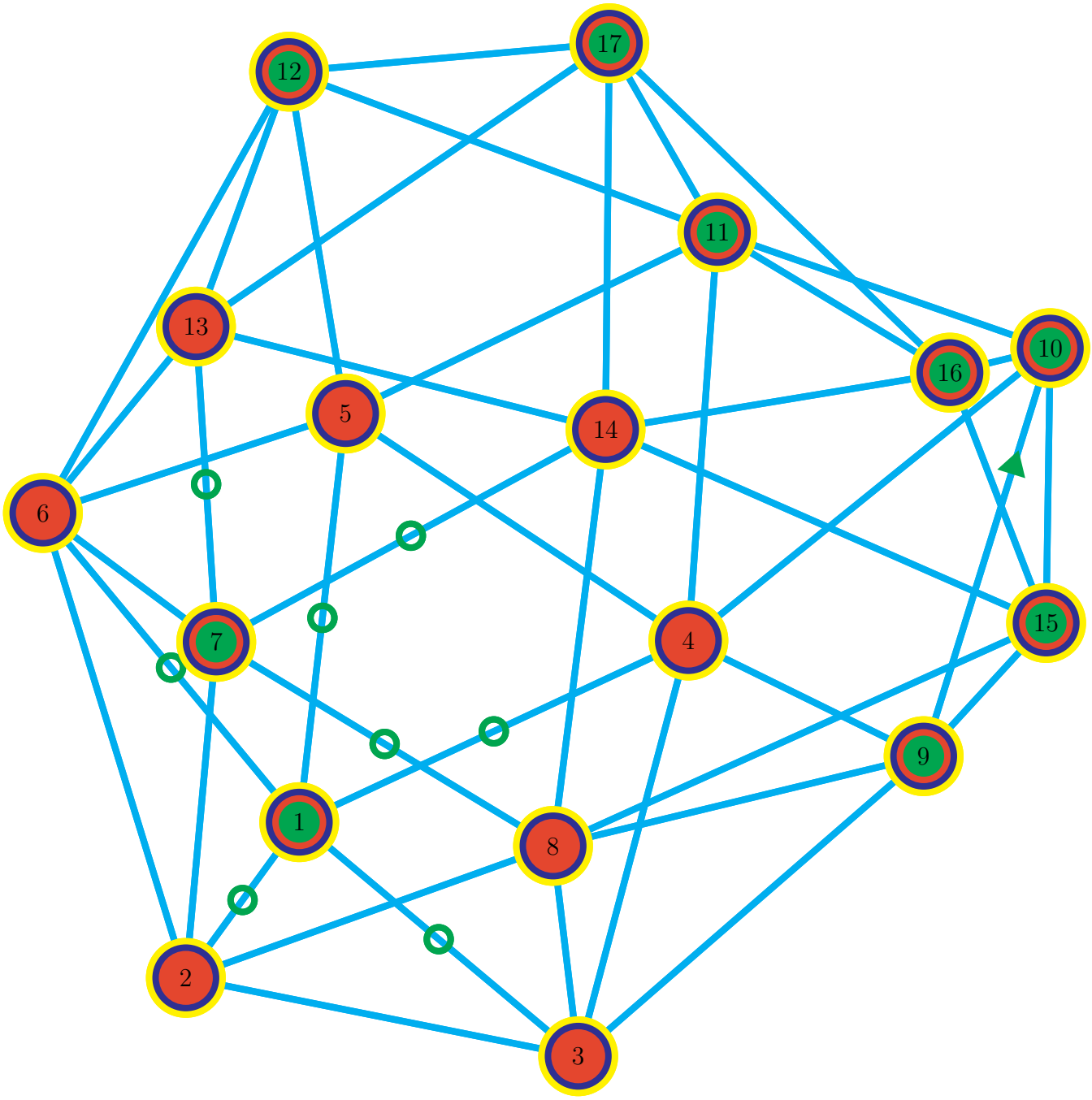

Figure 18: Instructions: 49: place edge 9→10 Green ArrowR

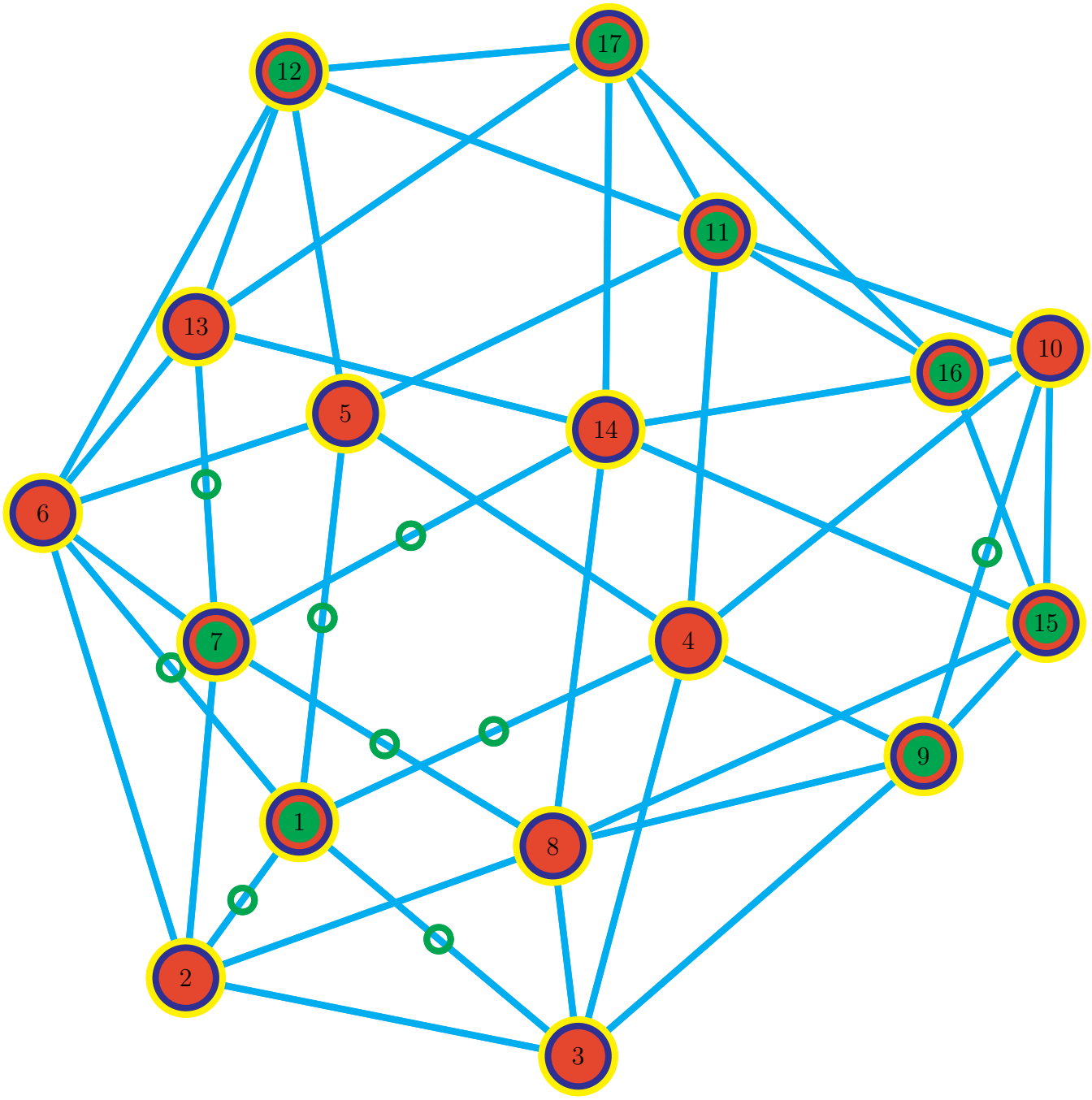

Figure 19: Instructions: 52: color edge 9→10 Green 53: uncolor vertex 10 Green

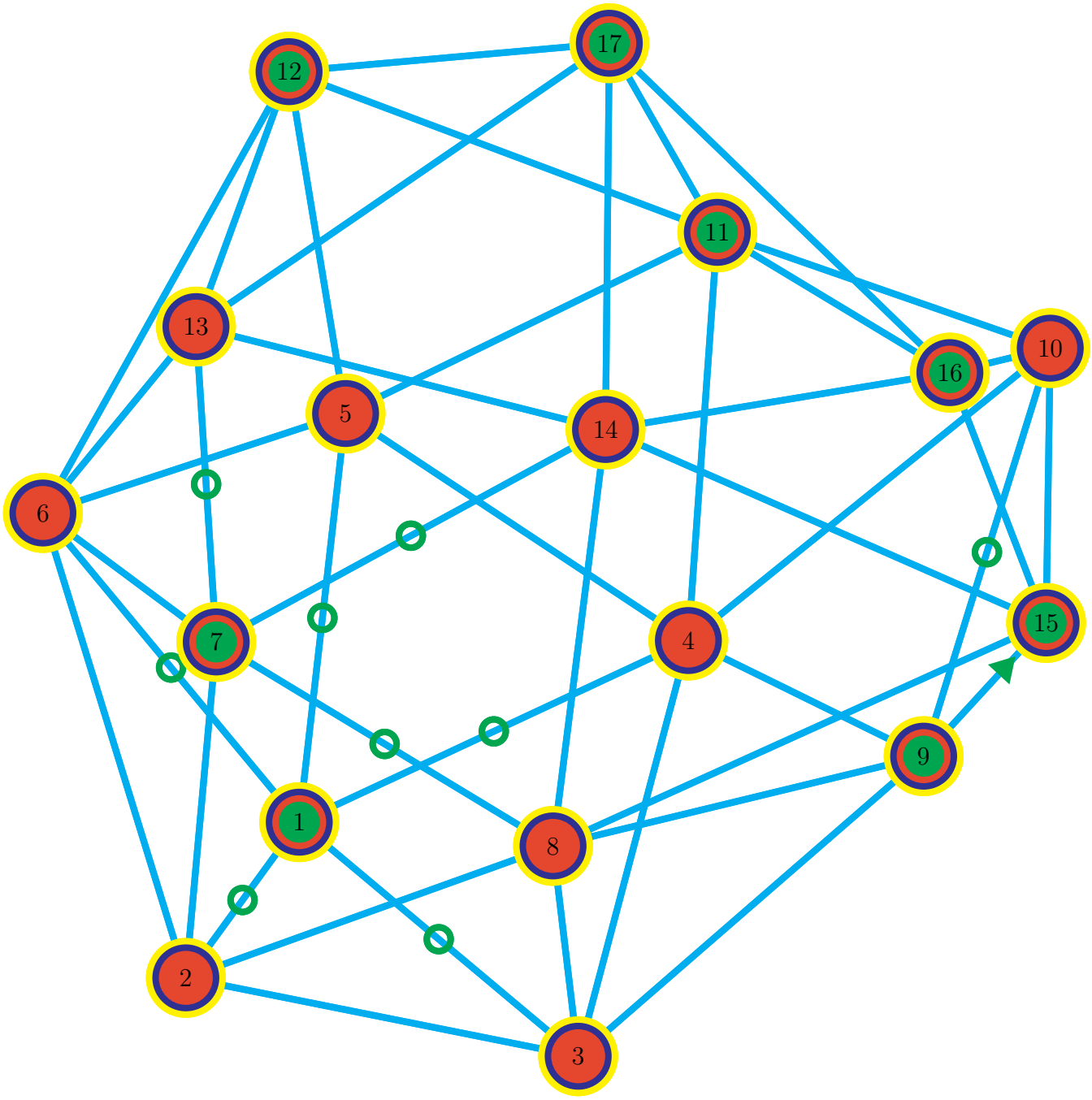

Figure 20: Instructions: 55: place edge 9→15 Green ArrowR

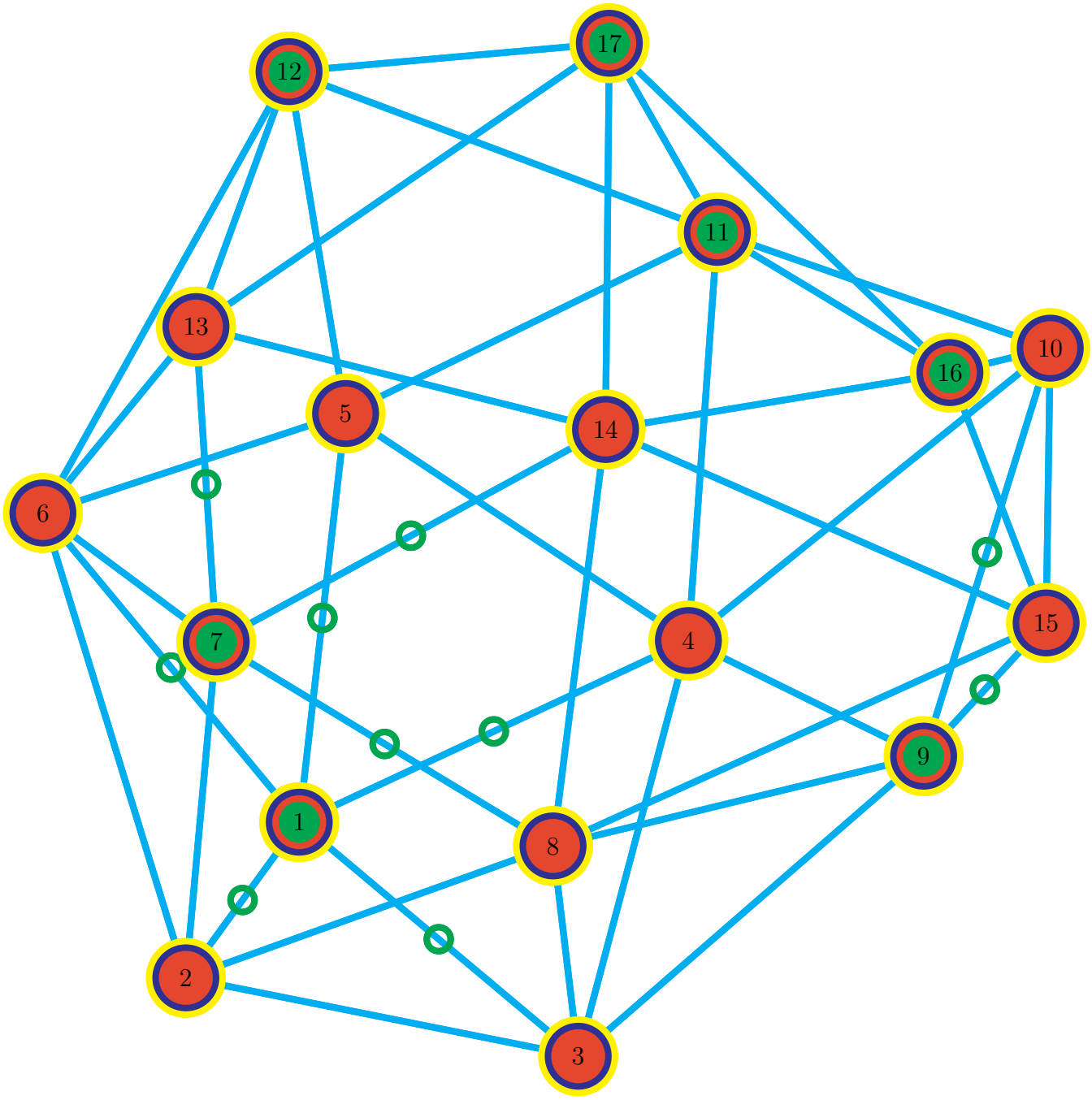

Figure 21: Instructions: 58: color edge 9→15 Green 59: uncolor vertex 15 Green

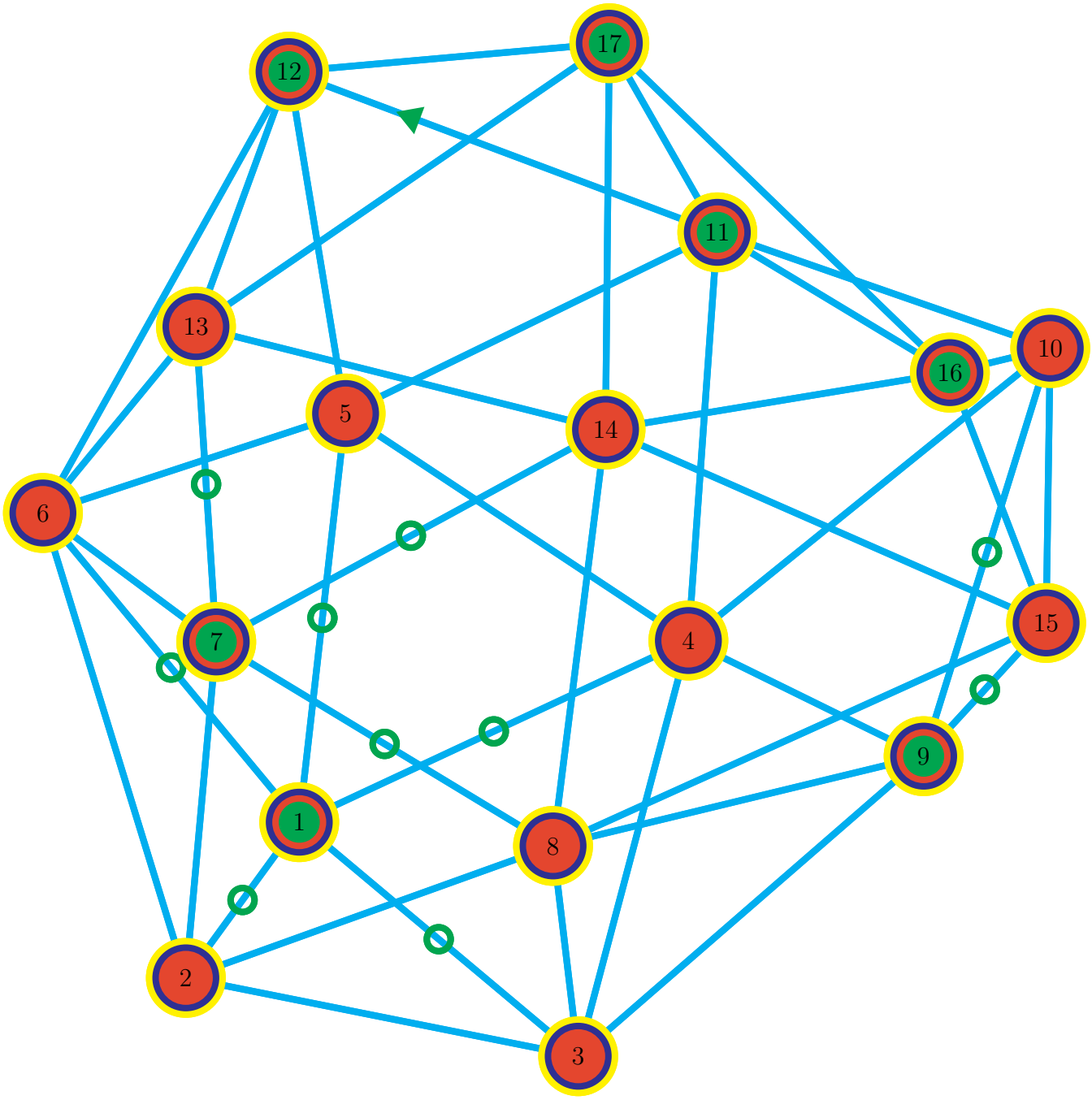

Figure 22: Instructions: 61: place edge 11→12 Green ArrowR

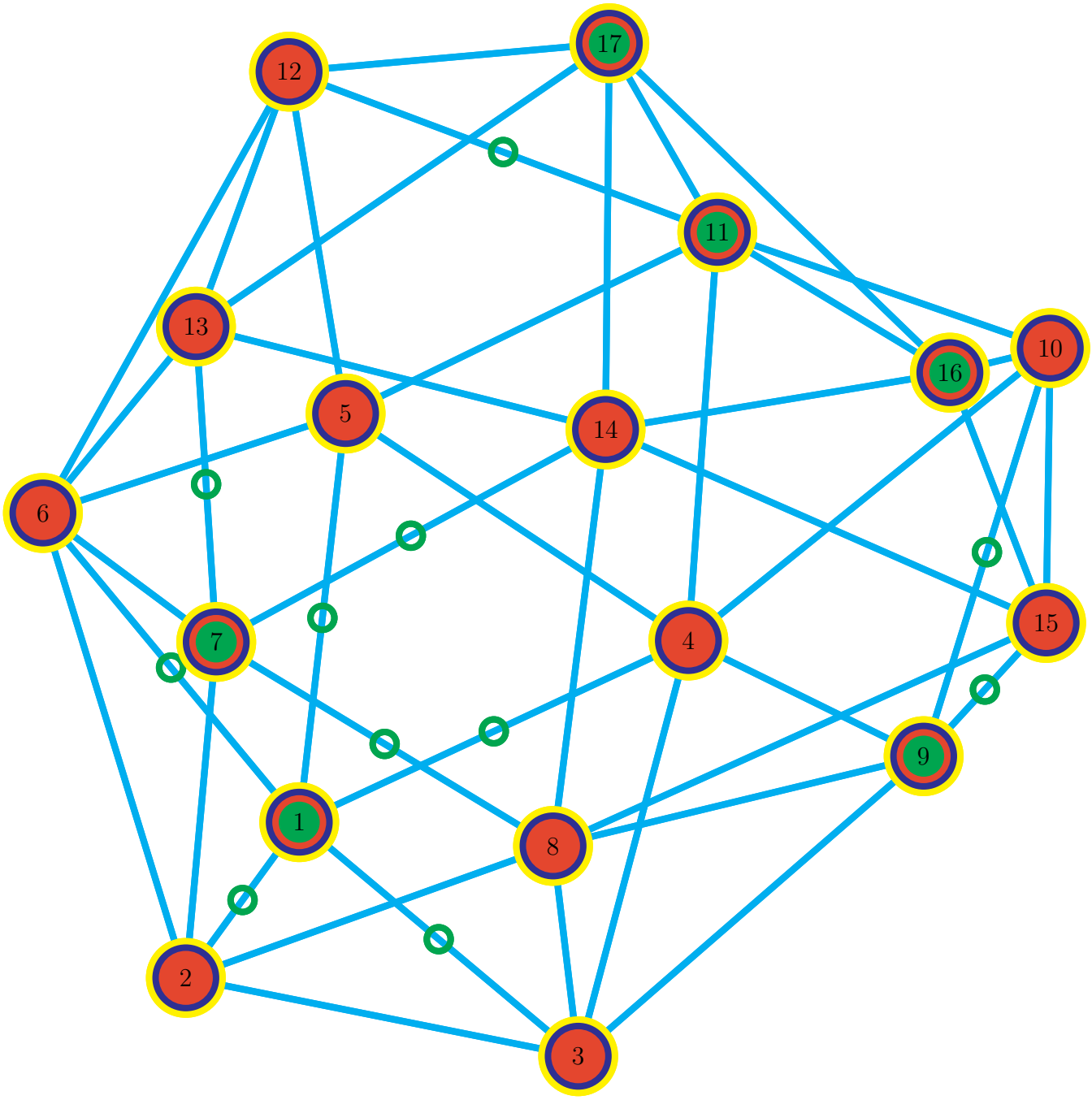

Figure 23: Instructions: 64: color edge 11→12 Green 65: uncolor vertex 12 Green

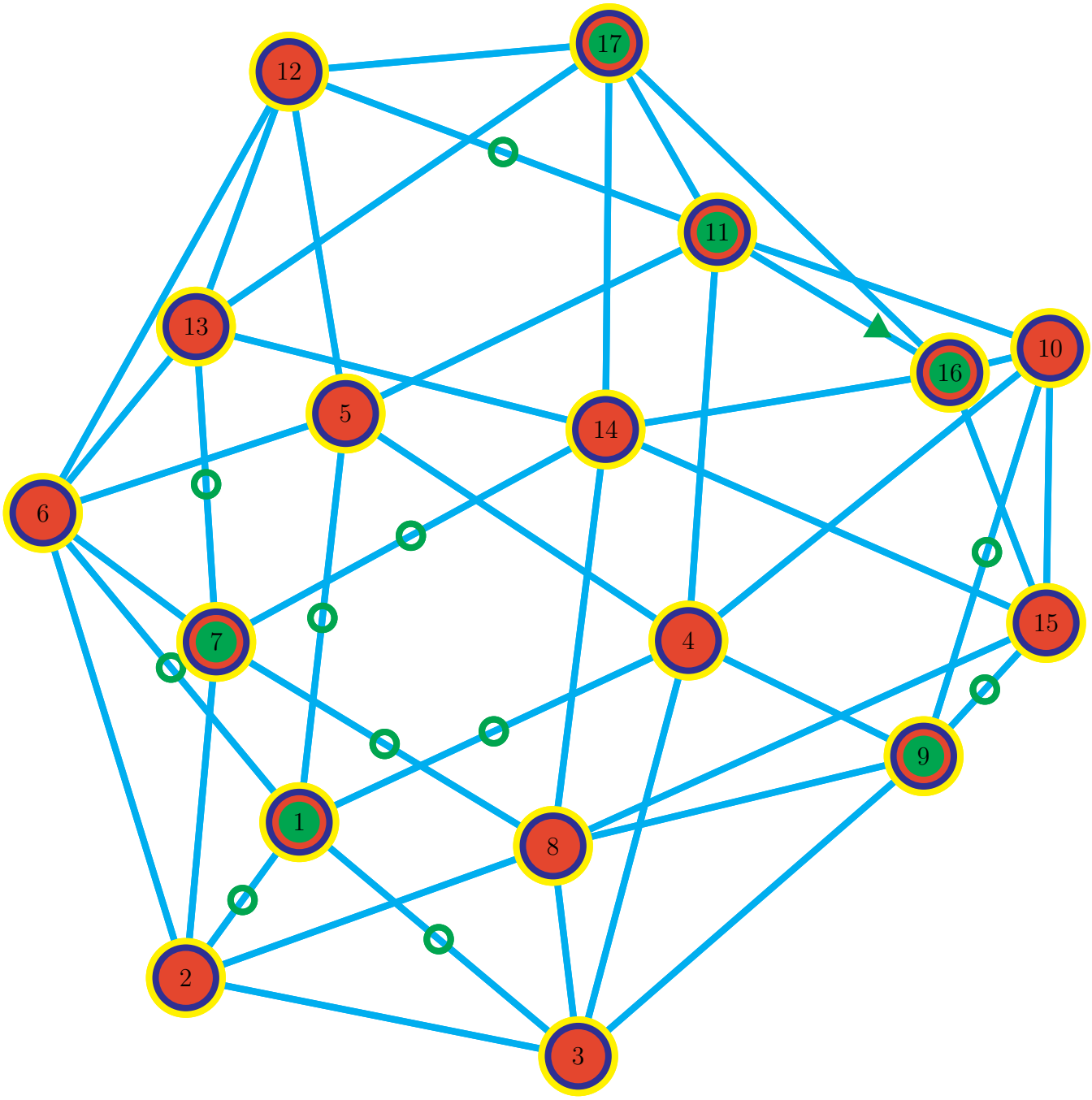

Figure 24: Instructions: 67: place edge 11→16 Green ArrowR

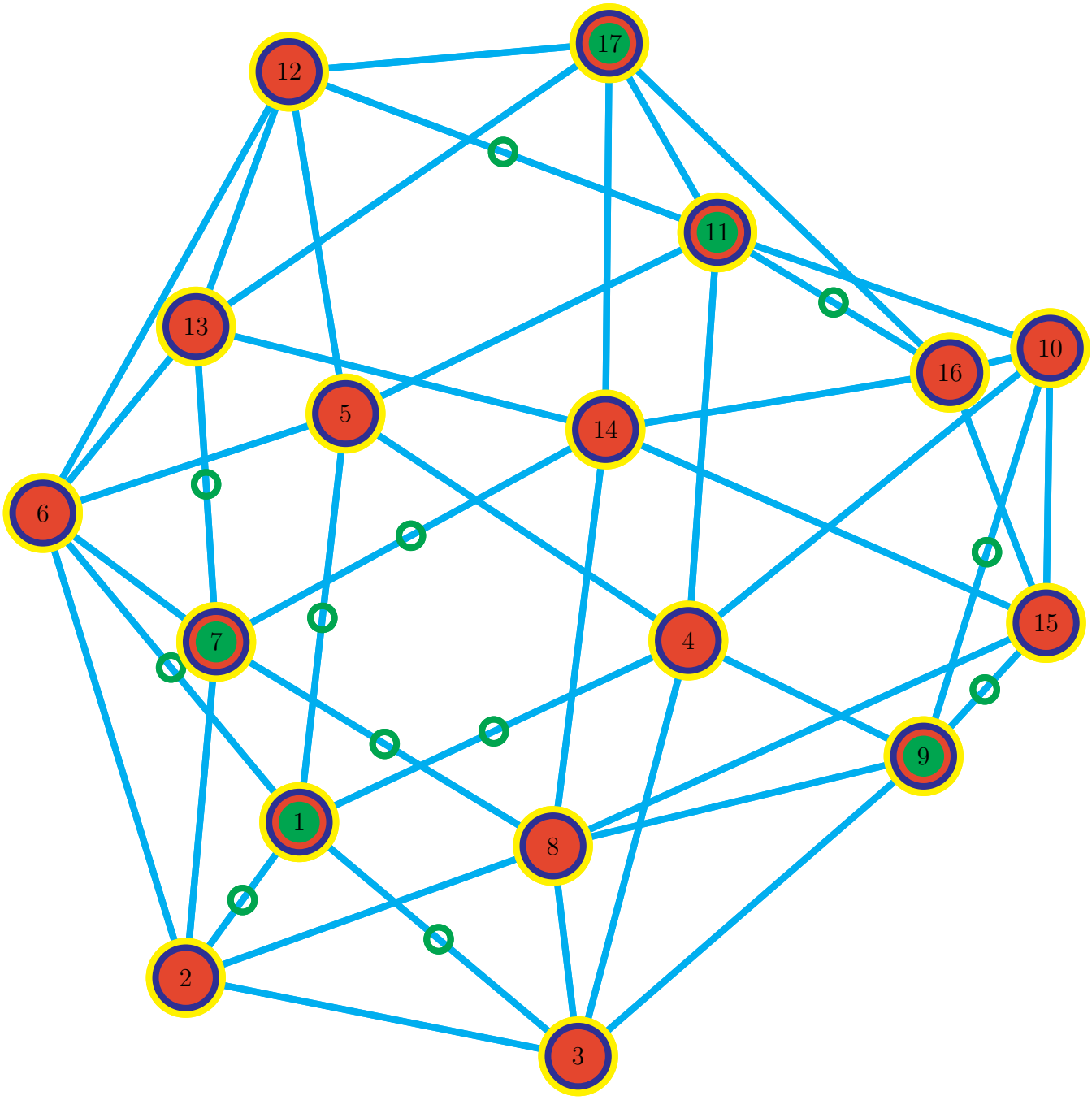

Figure 25: Instructions: 70: color edge 11→16 Green 71: uncolor vertex 16 Green

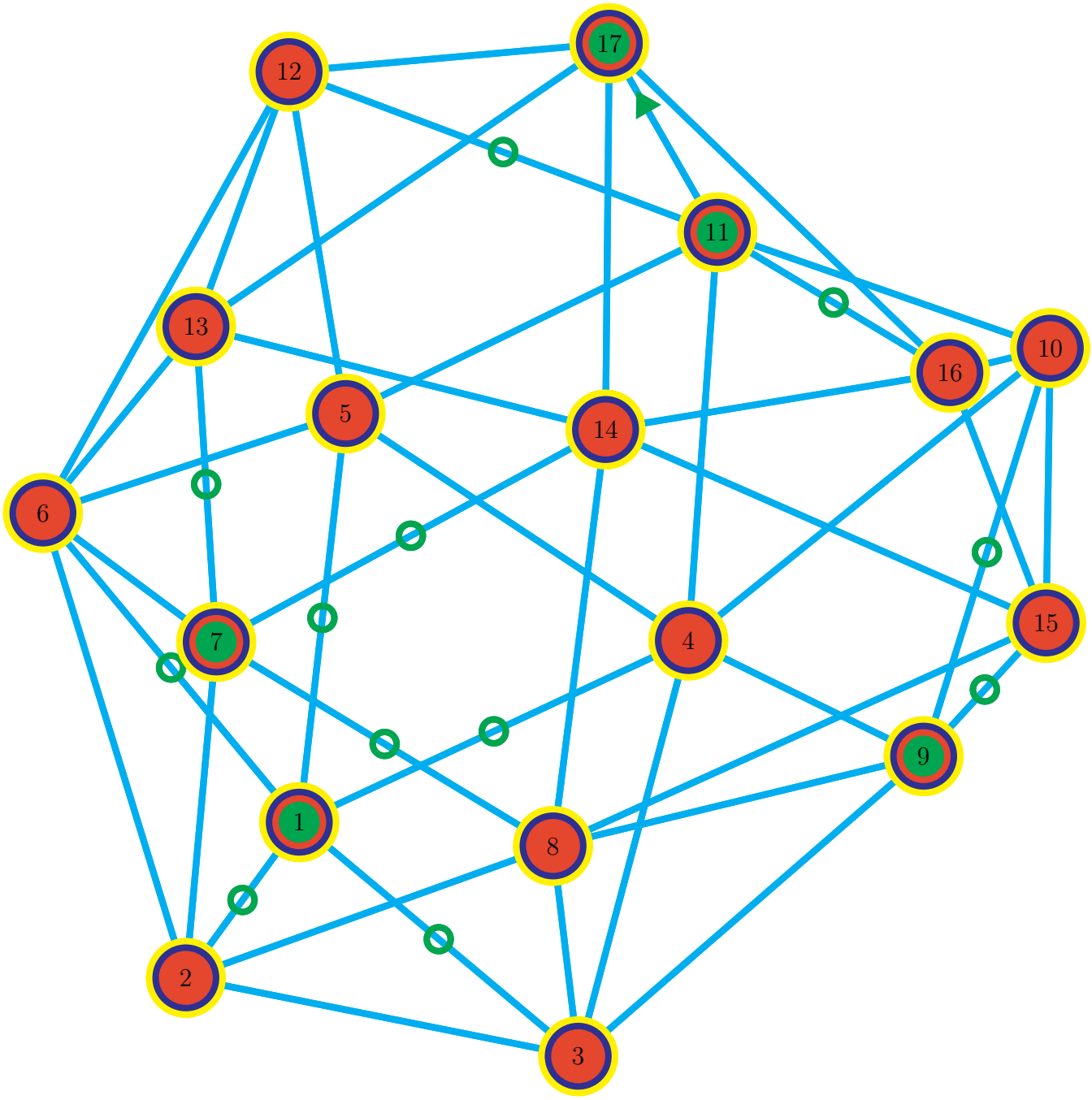

Figure 26: Instructions: 73: place edge 11→17 Green ArrowR

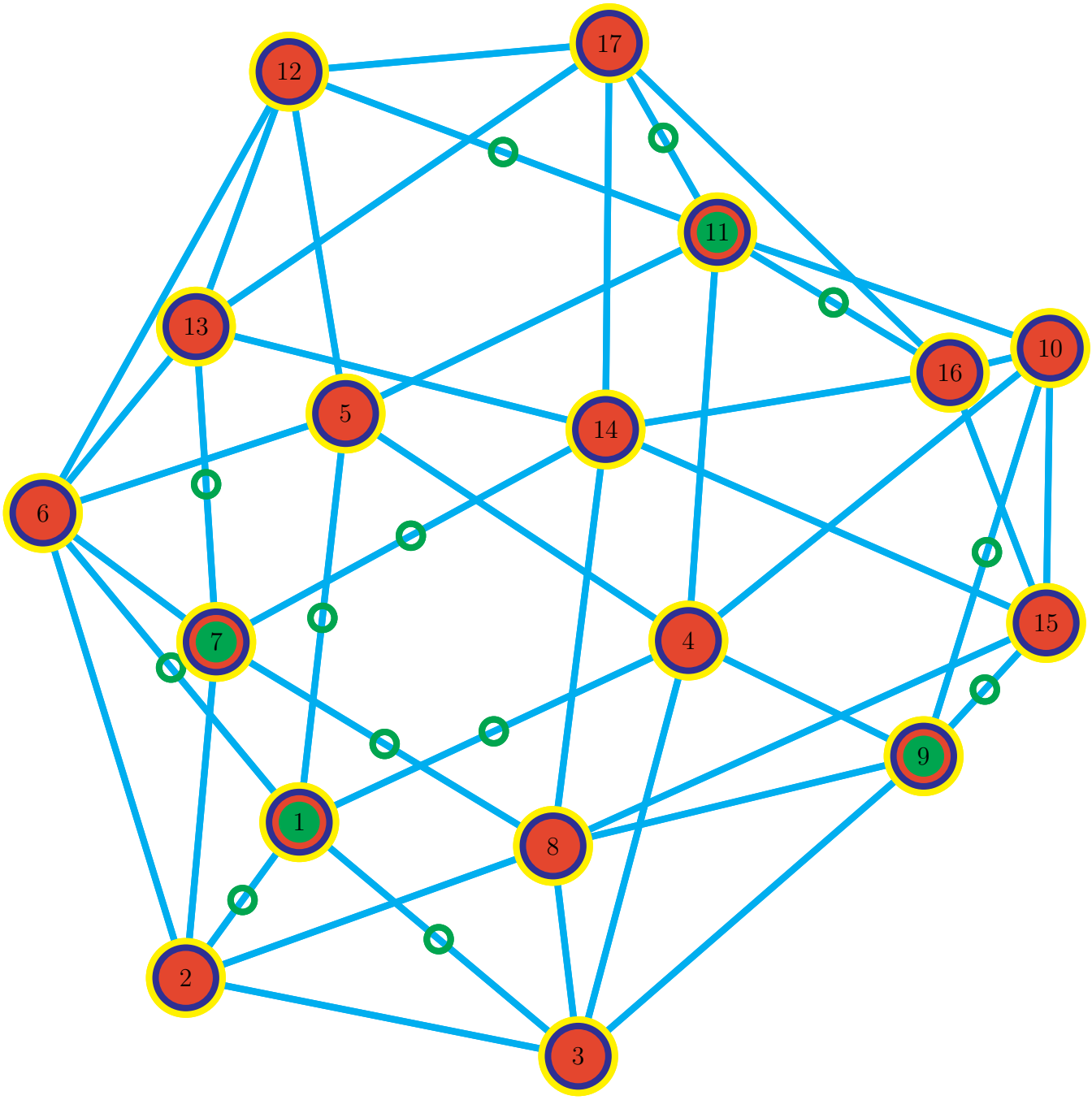

Figure 27: Instructions: 76: color edge 11→17 Green 77: uncolor vertex 17 Green

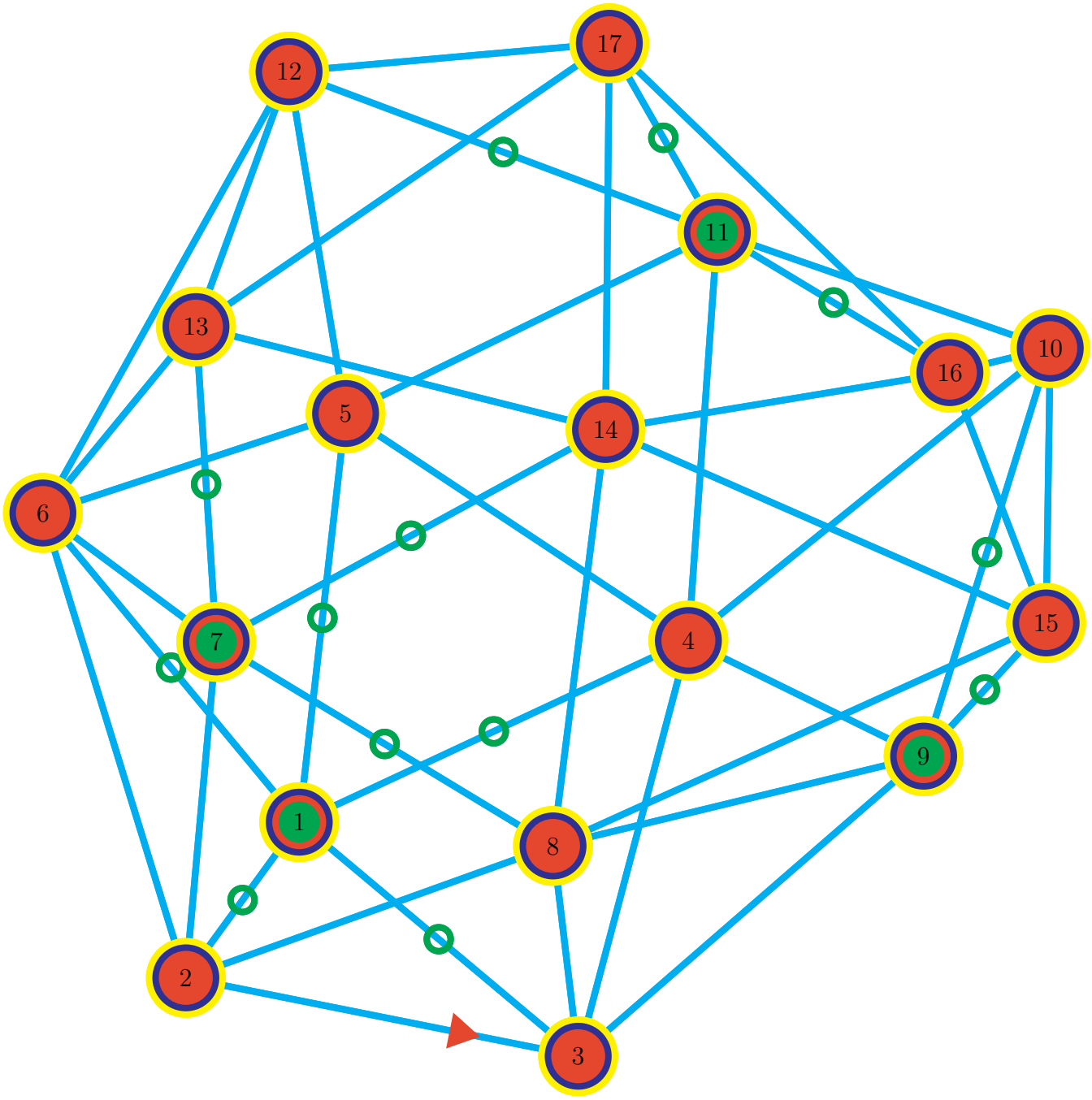

Figure 28: Instructions: 79: place edge 2→3 Red ArrowR

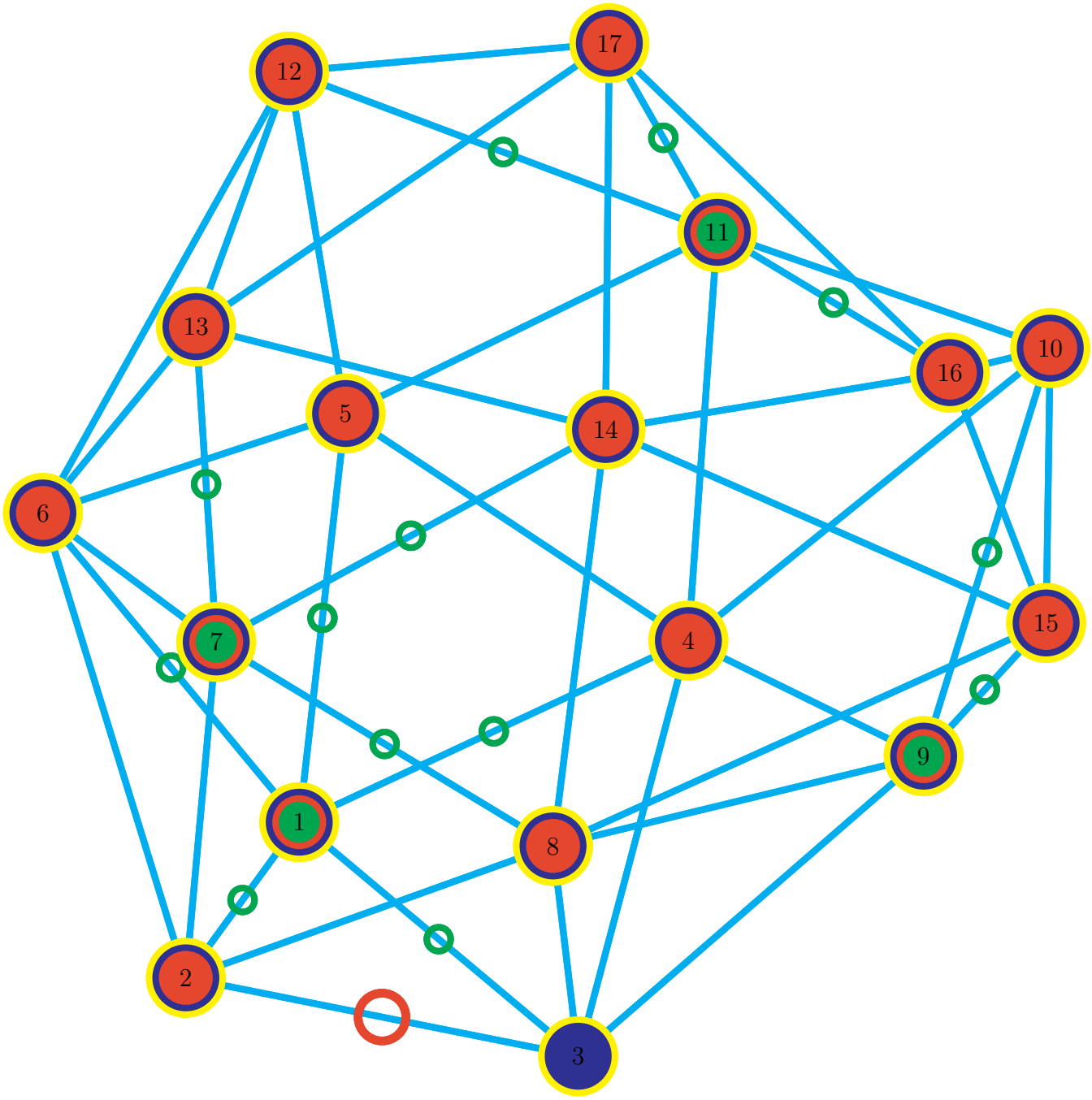

Figure 29: Instructions: 82: color edge 2→3 Red 83: uncolor vertex 3 Red

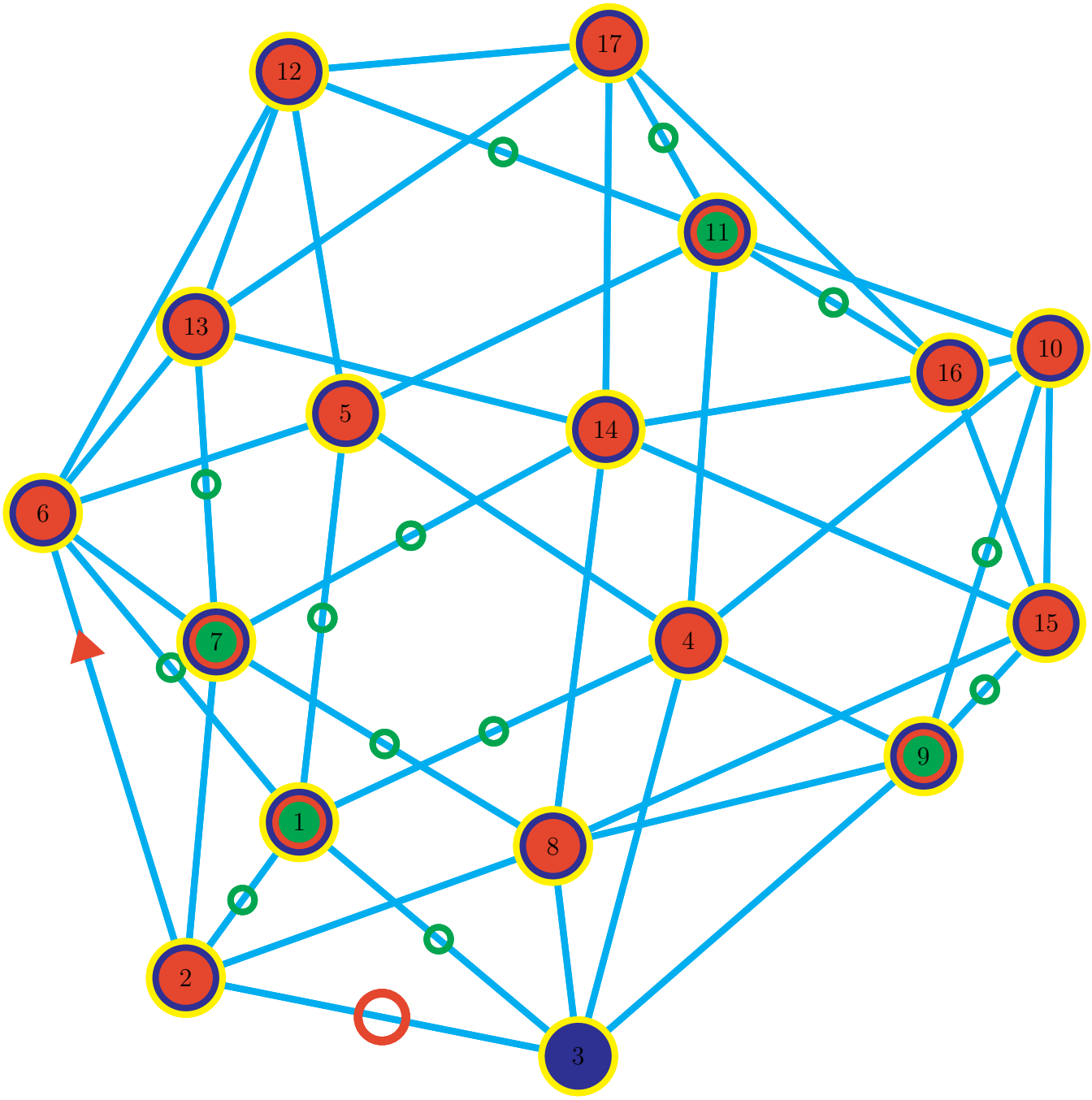

Figure 30: Instructions: 85: place edge 2→6 Red ArrowR

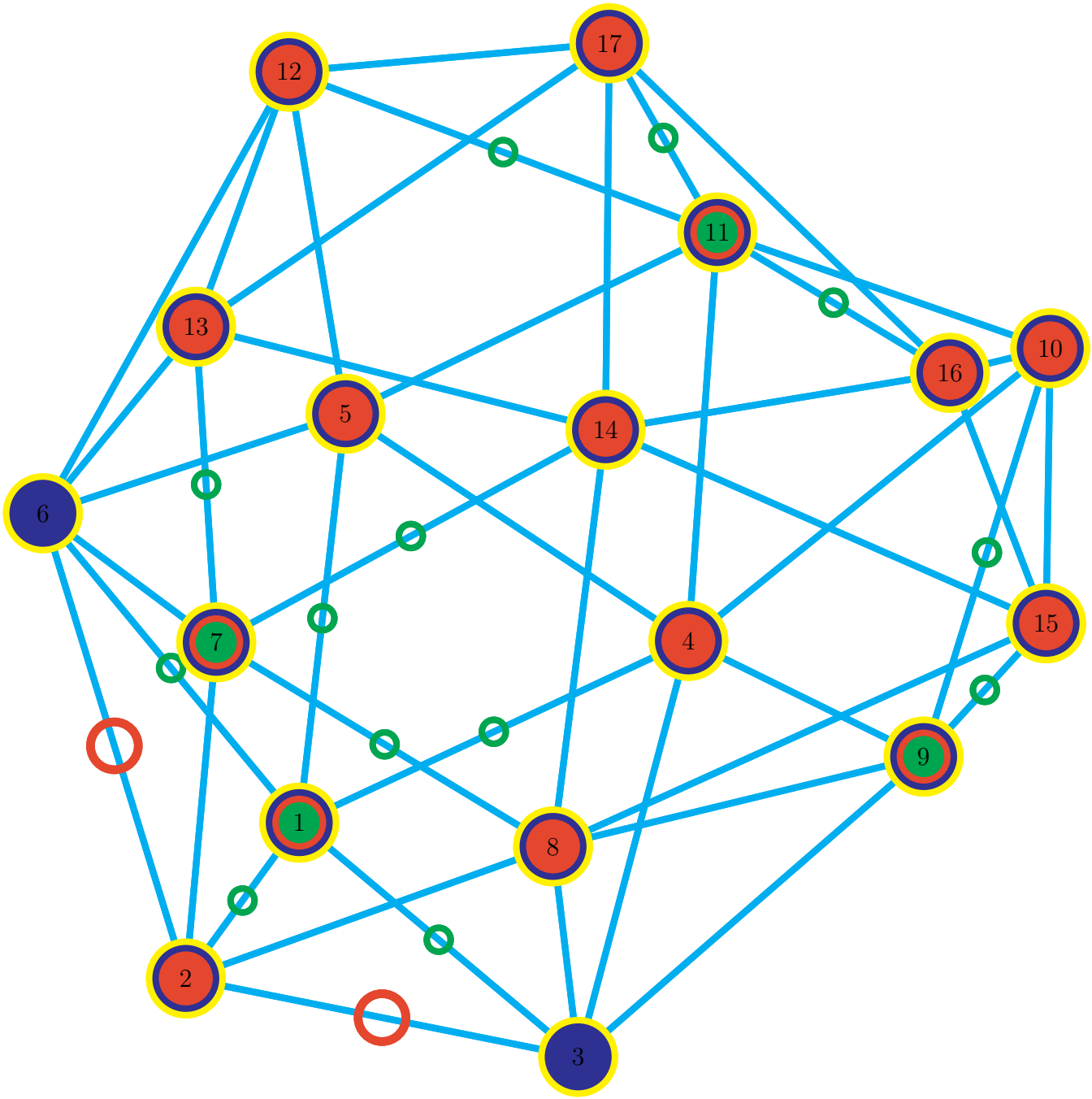

Figure 31: Instructions: 88: color edge 2→6 Red 89: uncolor vertex 6 Red

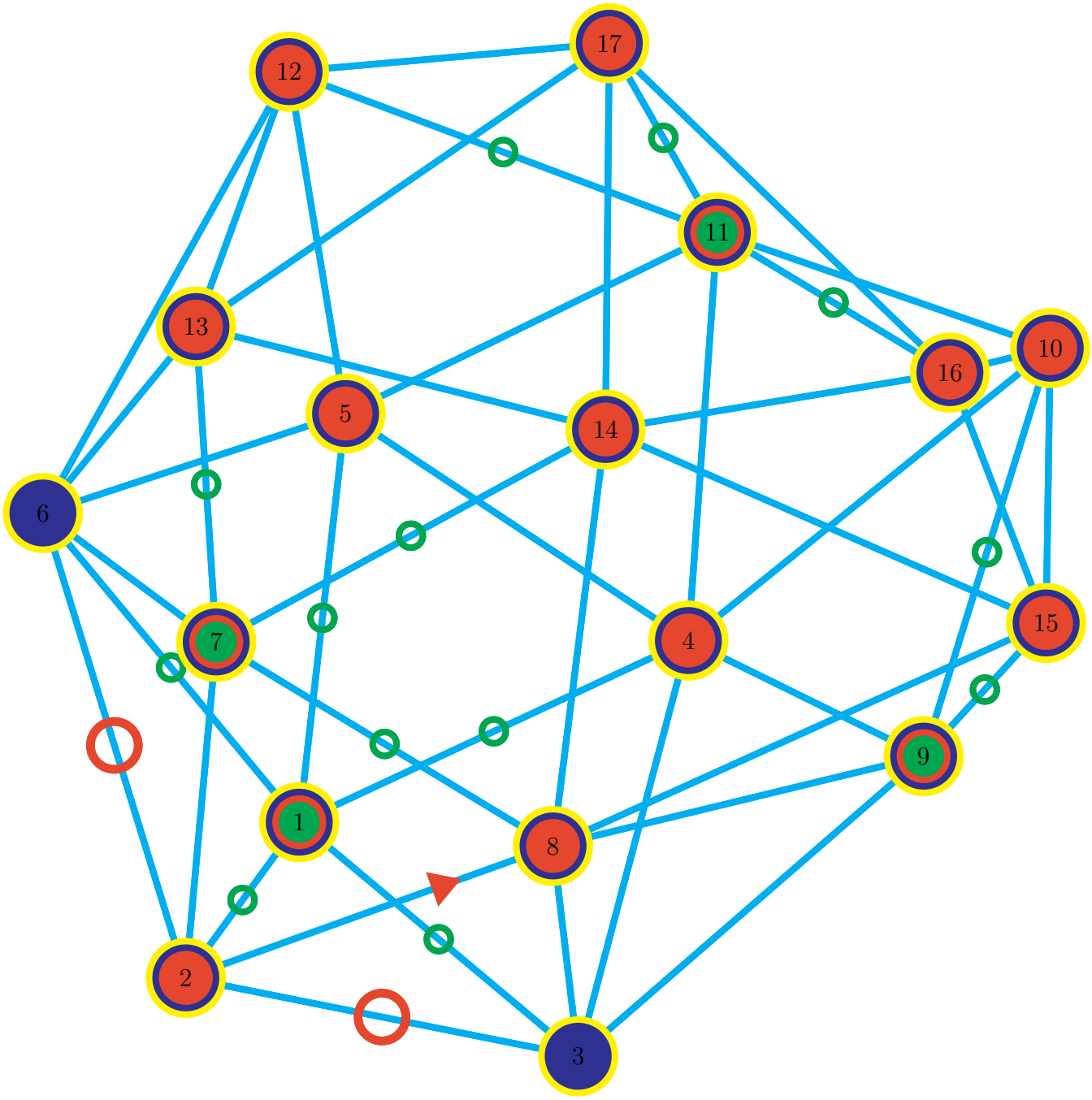

Figure 32: Instructions: 91: place edge 2→8 Red ArrowR

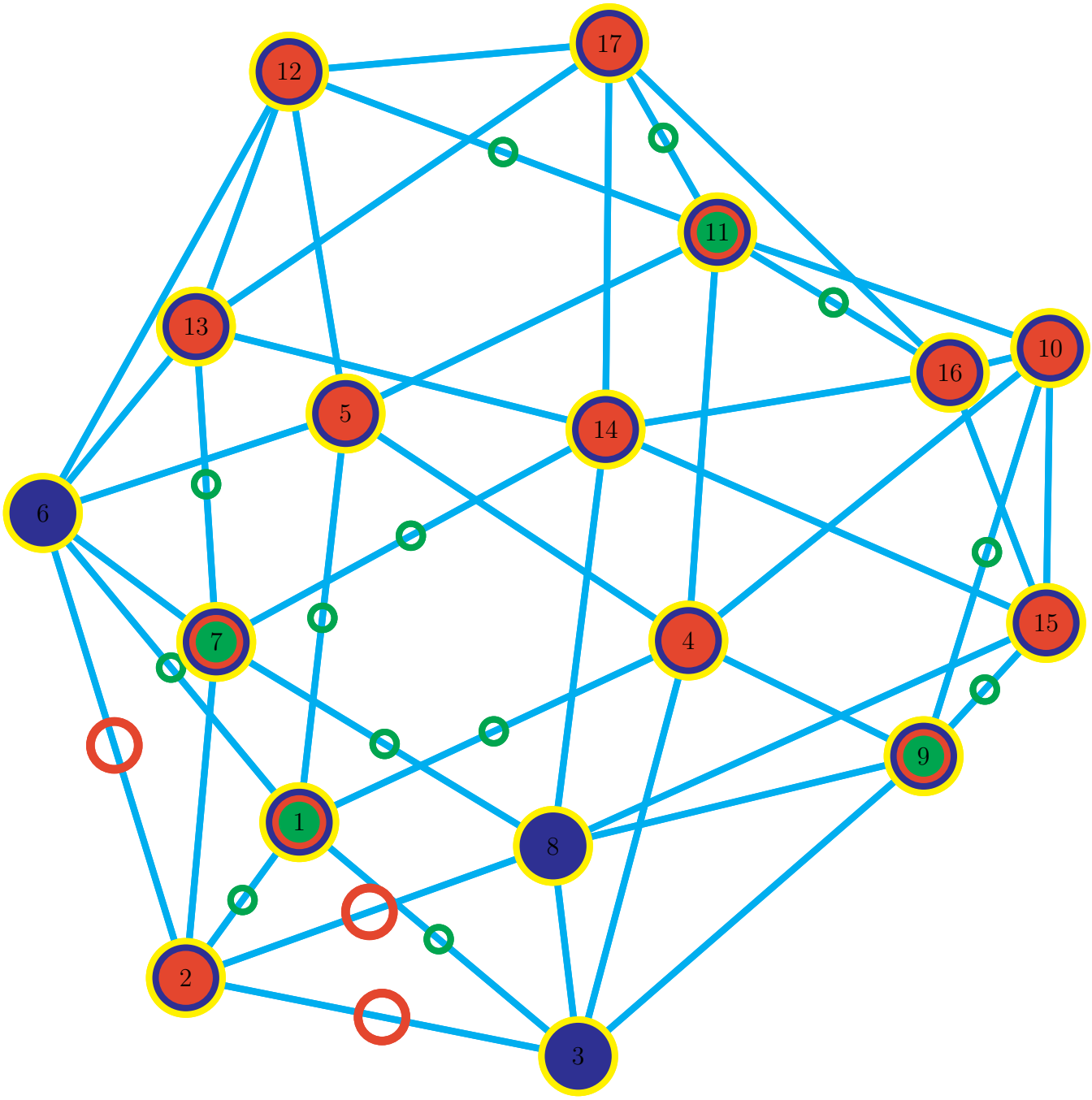

Figure 33: Instructions: 94: color edge 2→8 Red 95: uncolor vertex 8 Red

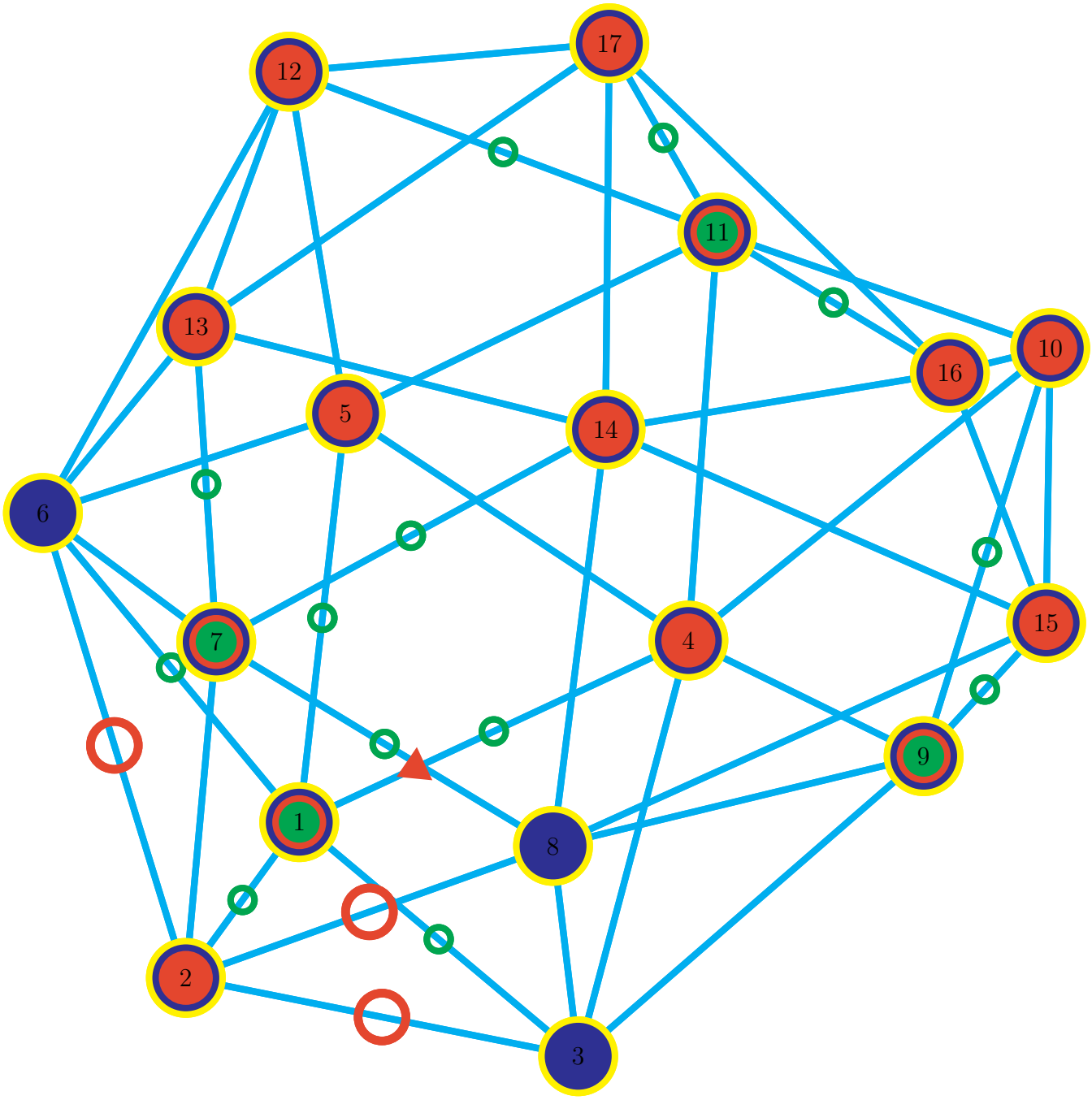

Figure 34: Instructions: 97: place edge 4→1 Red ArrowR

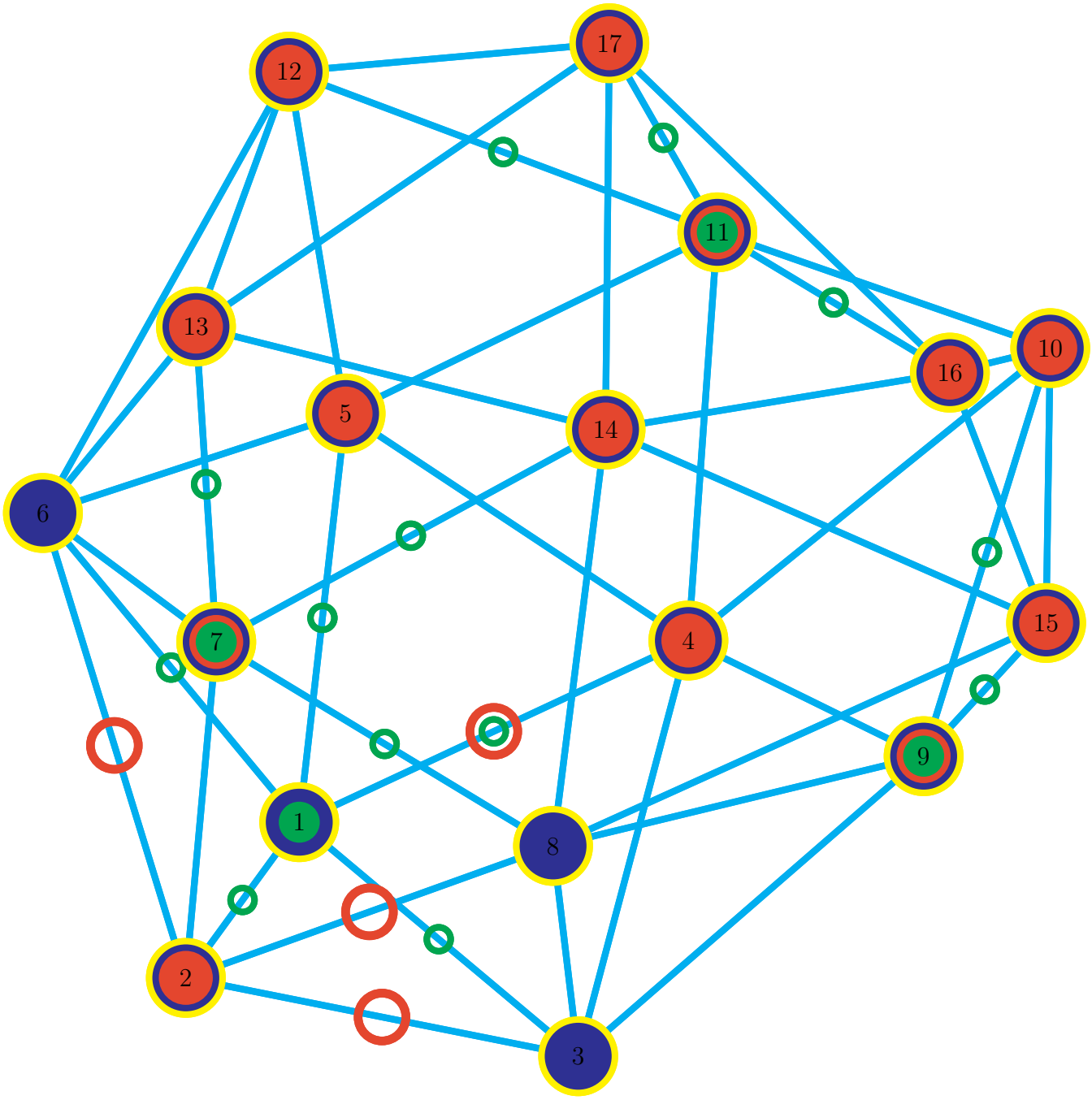

Figure 35: Instructions: 100: color edge 4→1 Red 101: uncolor vertex 1 Red

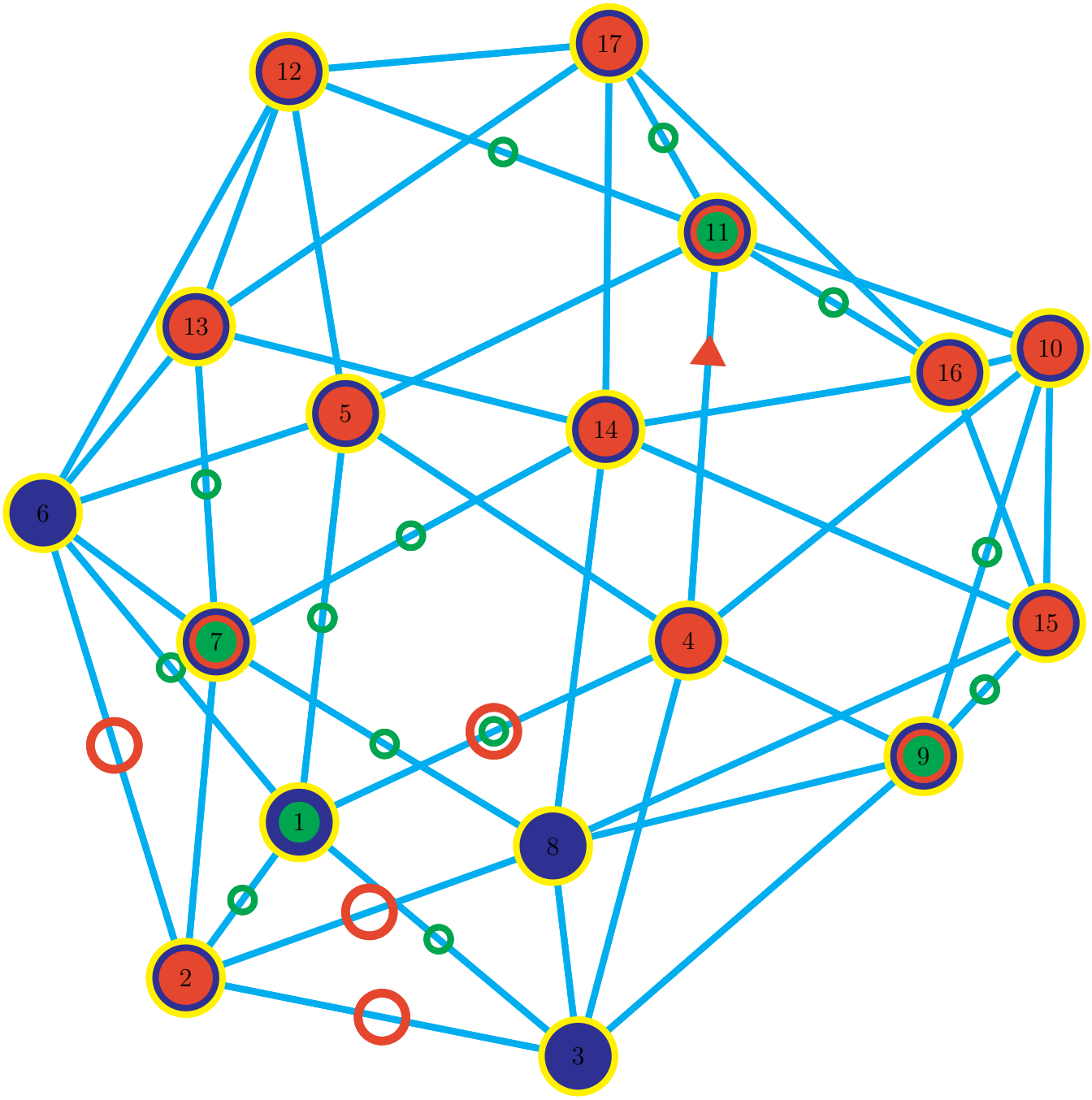

Figure 36: Instructions: 103: place edge 4→11 Red ArrowR

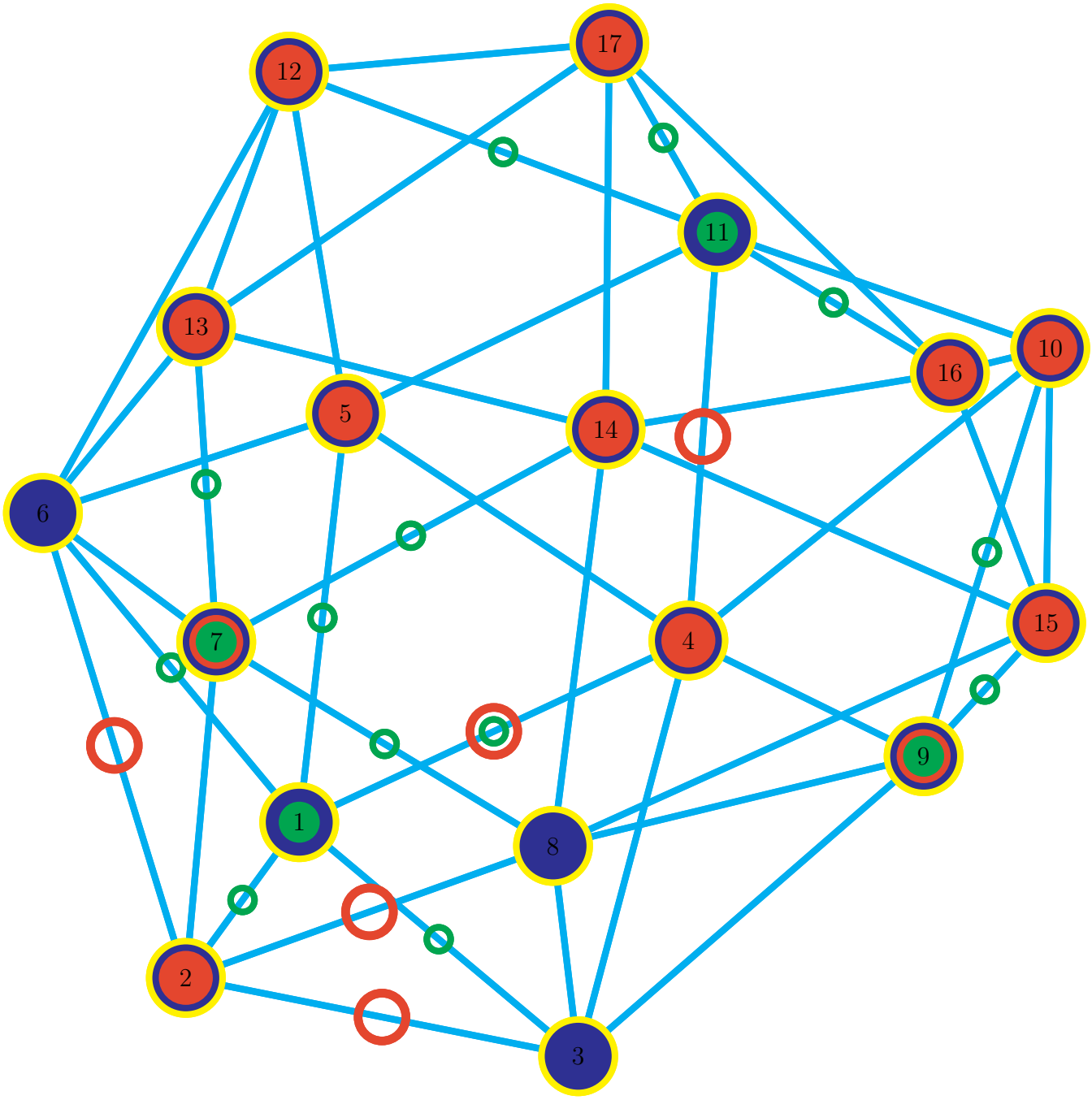

Figure 37: Instructions: 106: color edge 4→11 Red 107: uncolor vertex 11 Red

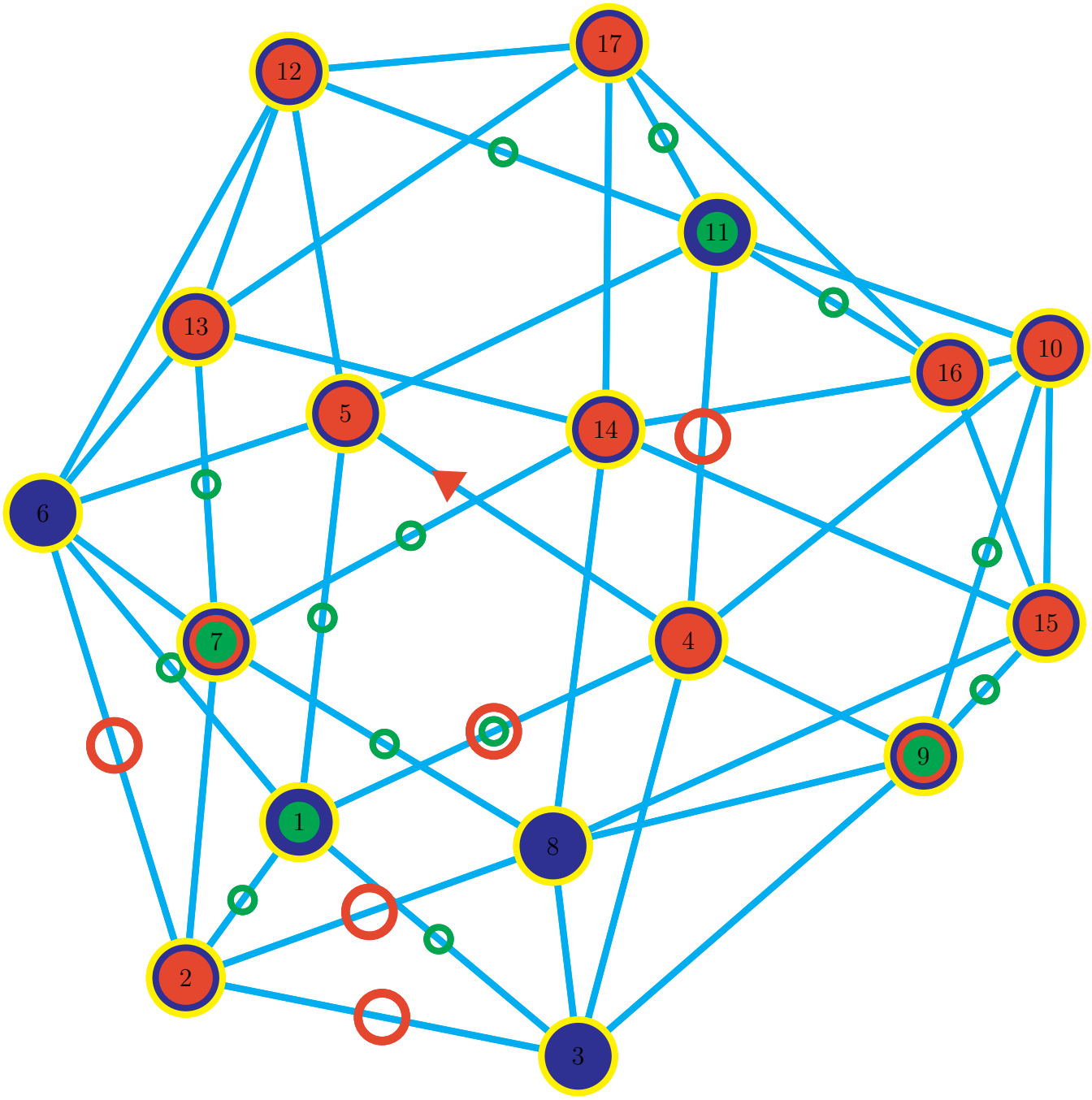

Figure 38: Instructions: 109: place edge 4→5 Red ArrowR

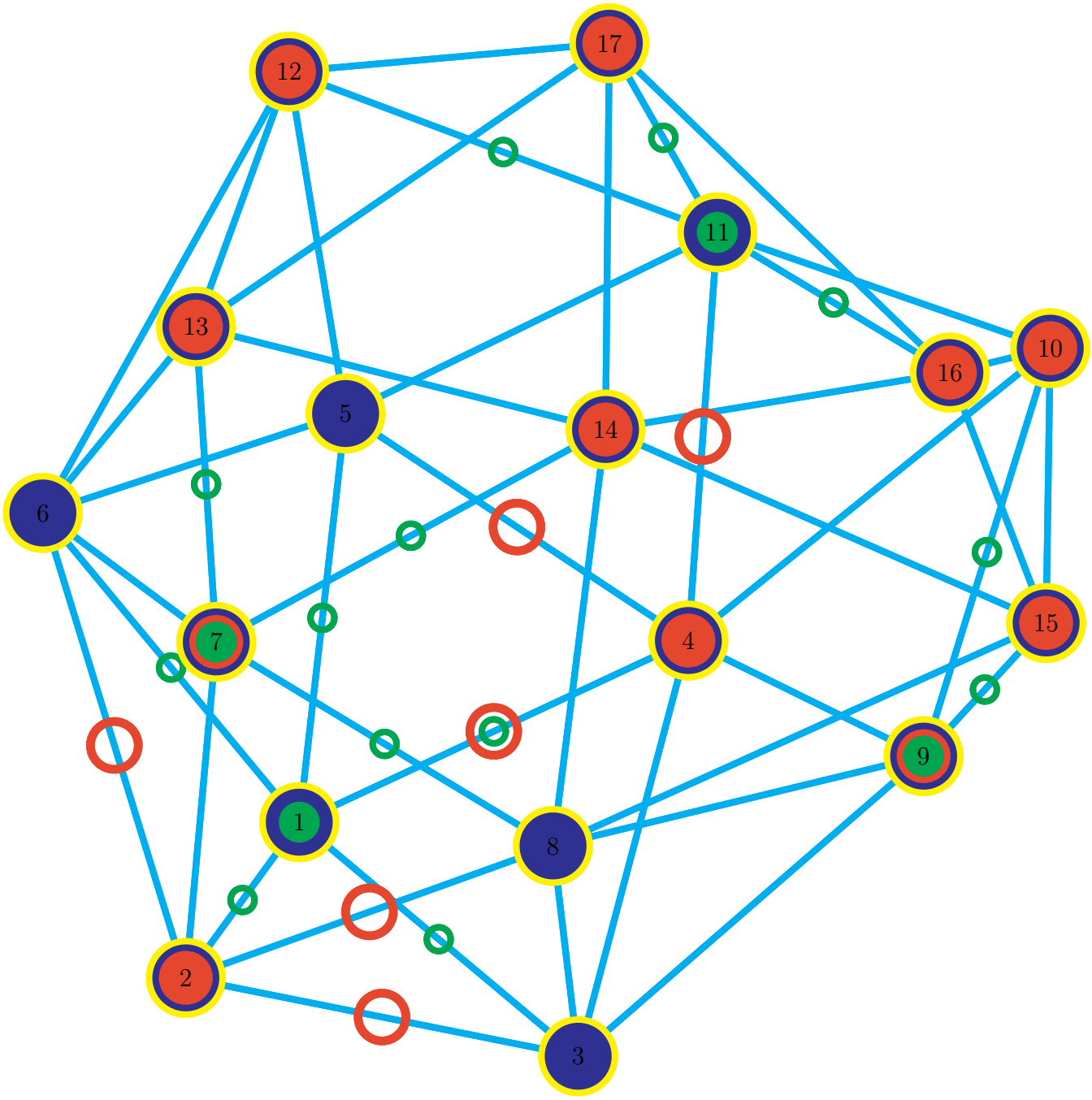

Figure 39: Instructions: 112: color edge 4→5 Red 113: uncolor vertex 5 Red

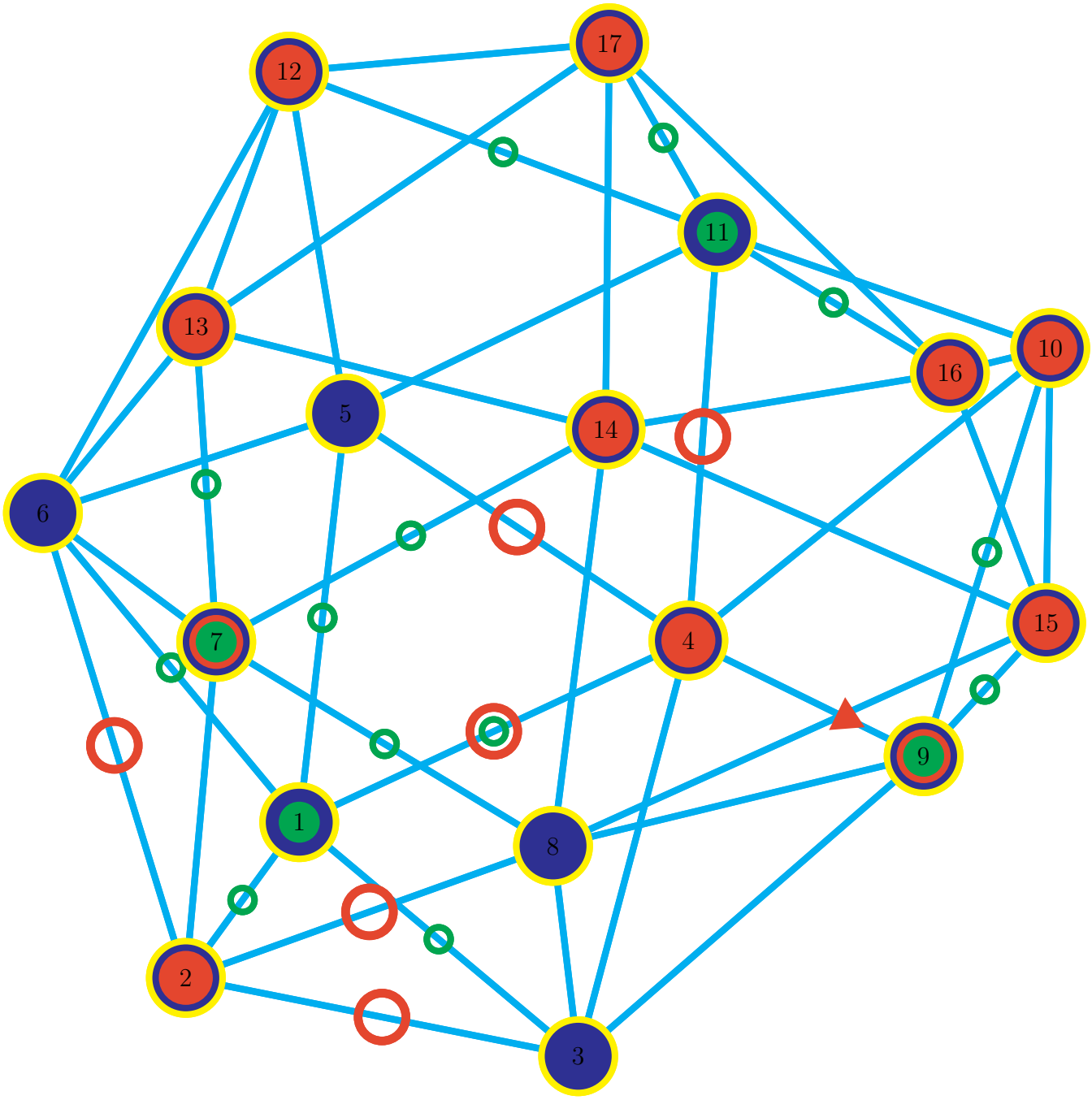

Figure 40: Instructions: 115: place edge 4→9 Red ArrowR

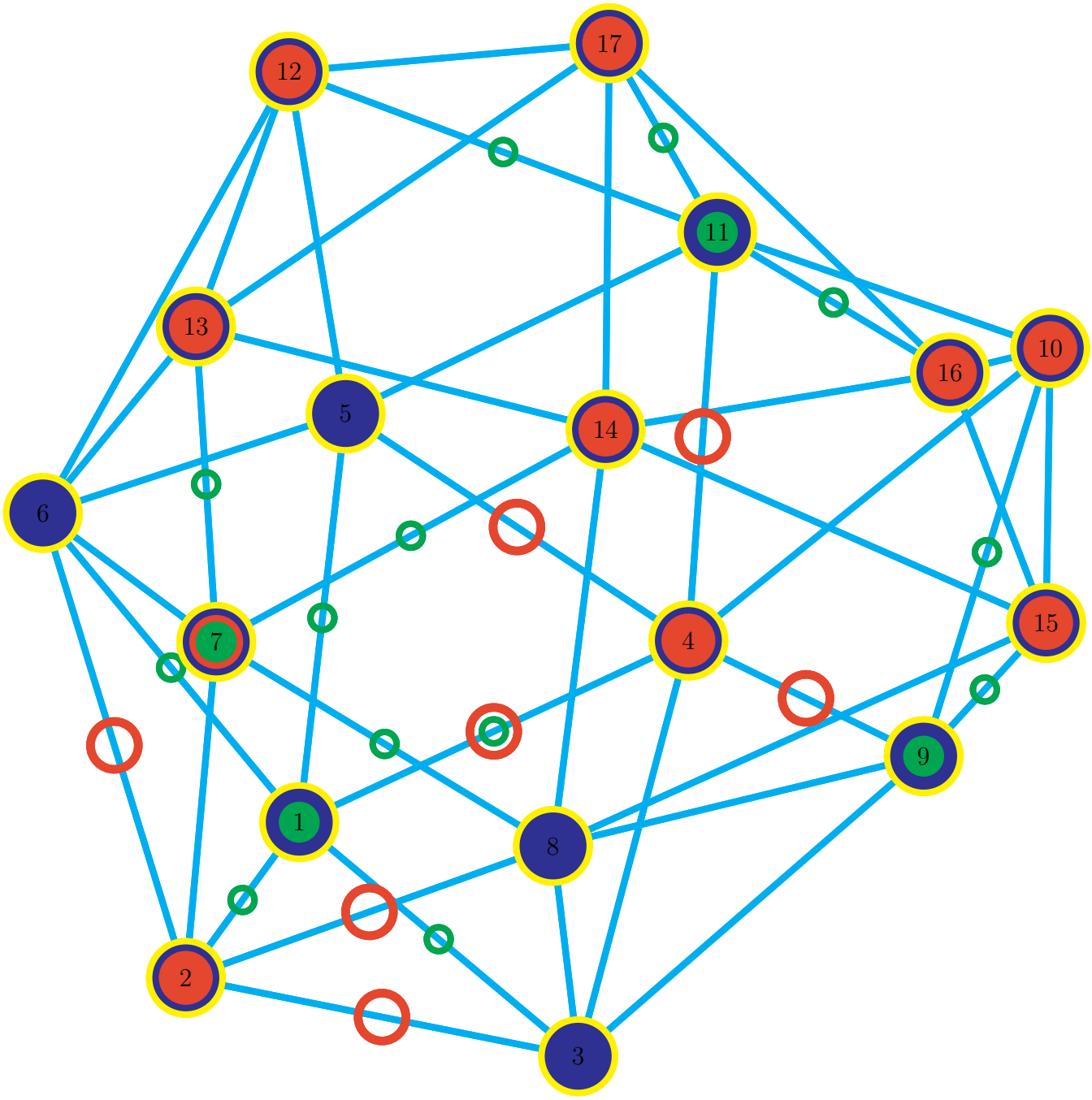

Figure 41: Instructions: 118: color edge 4→9 Red 119: uncolor vertex 9 Red

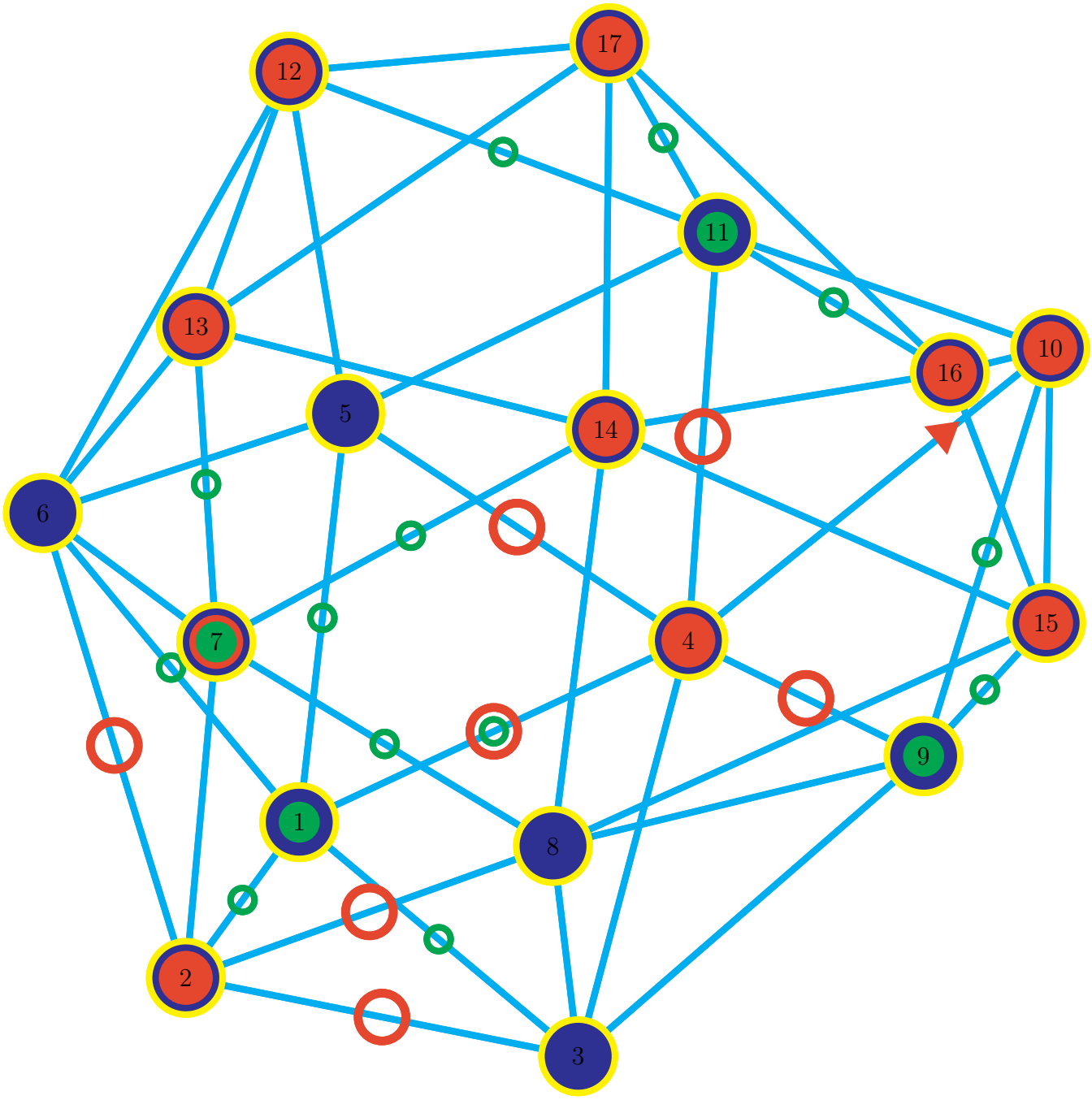

Figure 42: Instructions: 121: place edge 4→10 Red ArrowR

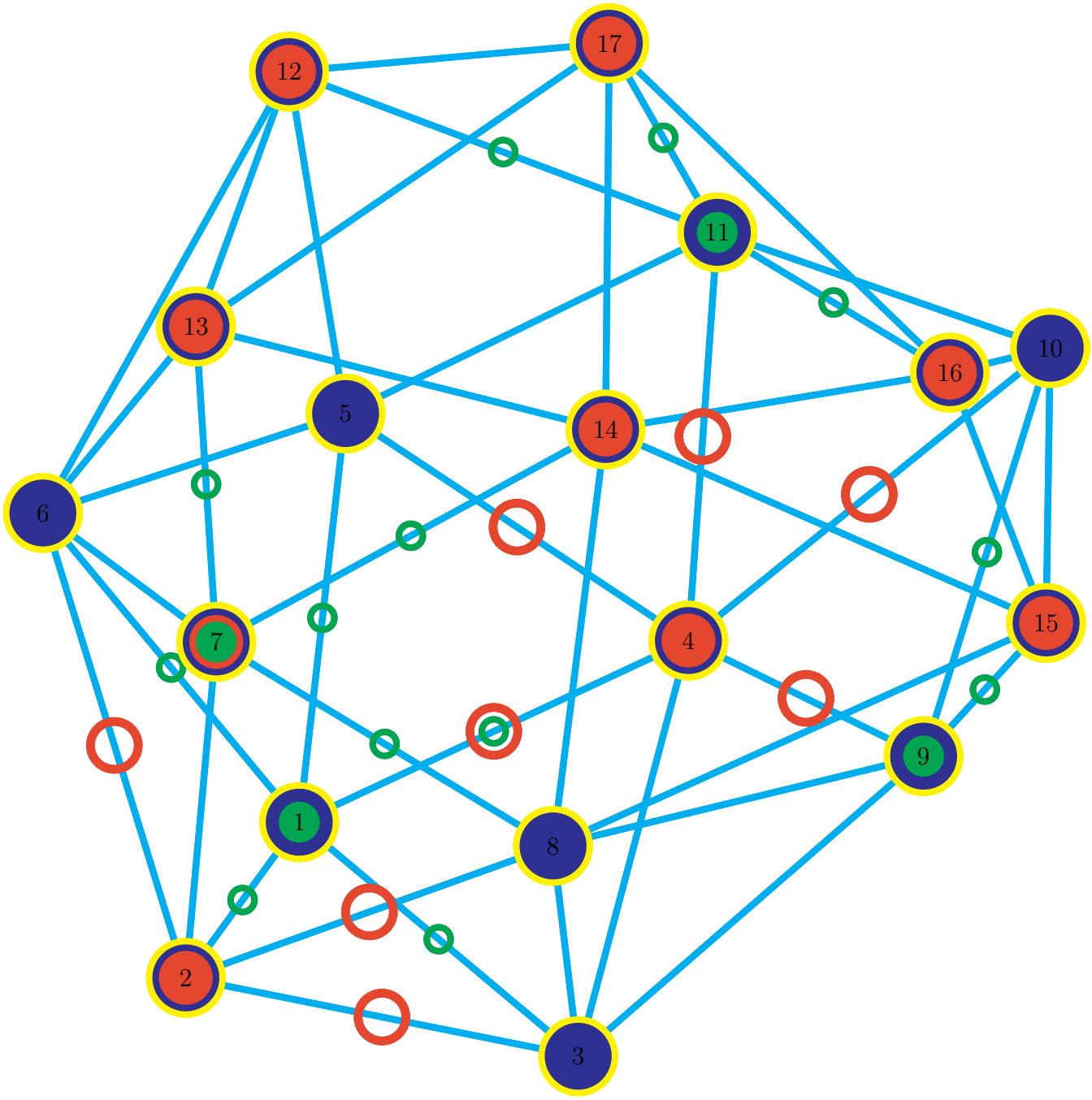

Figure 43: Instructions: 124: color edge 4→10 Red 125: uncolor vertex 10 Red

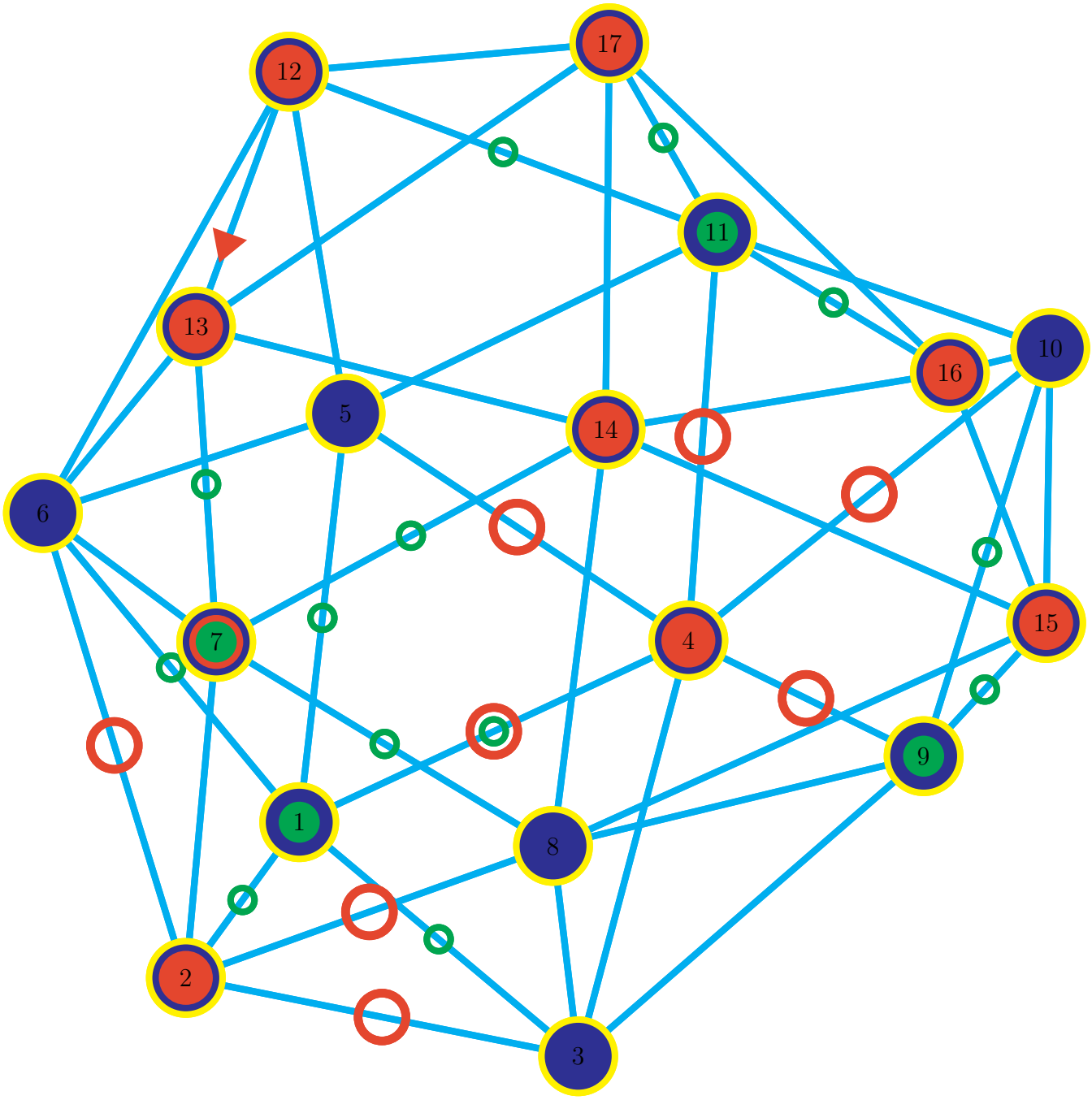

Figure 44: Instructions: 127: place edge 12→13 Red ArrowR

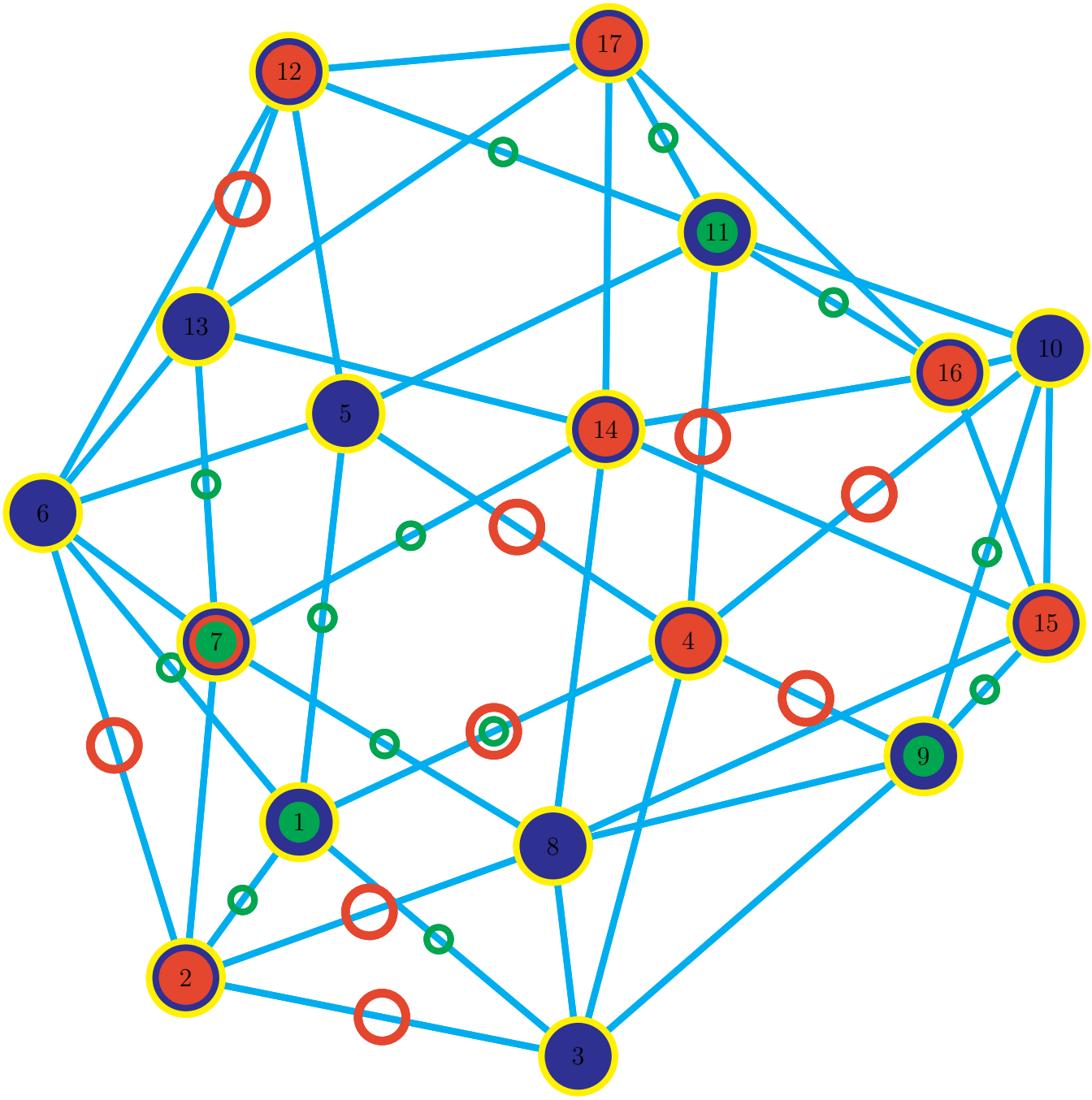

Figure 45: Instructions: 130: color edge 12→13 Red 131: uncolor vertex 13 Red

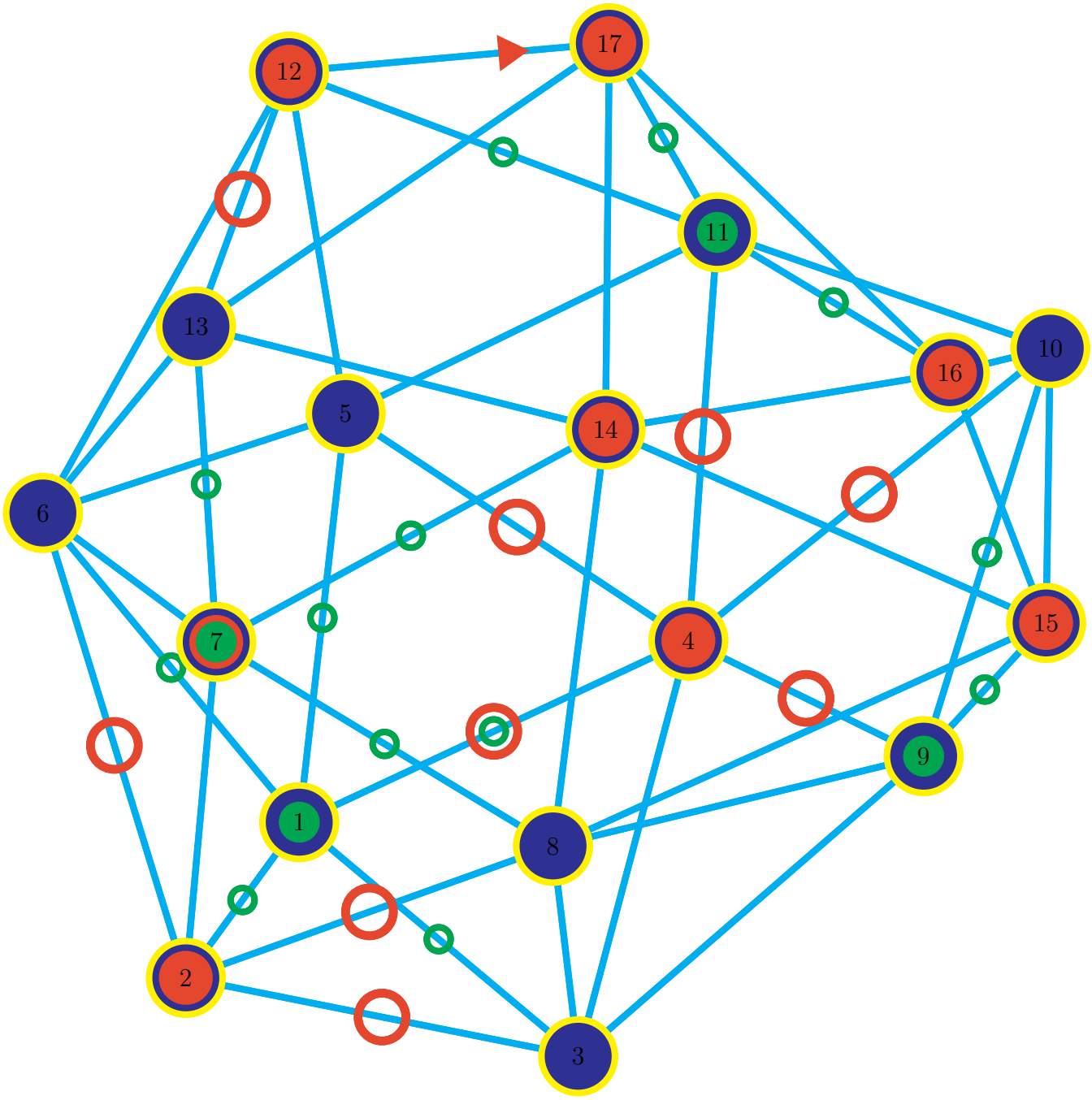

Figure 46: Instructions: 133: place edge 12→17 Red ArrowR

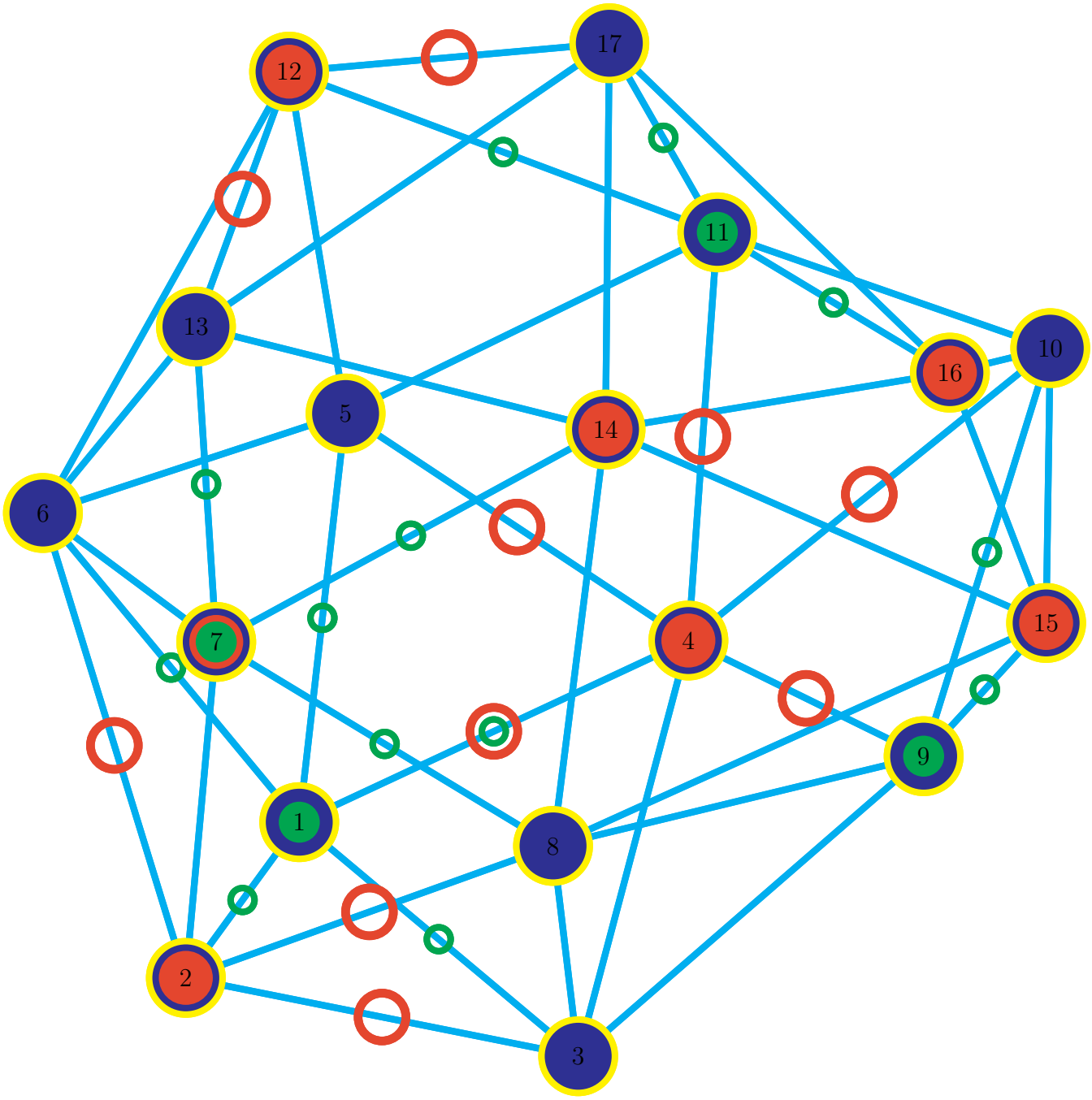

Figure 47: Instructions: 136: color edge 12→17 Red 137: uncolor vertex 17 Red

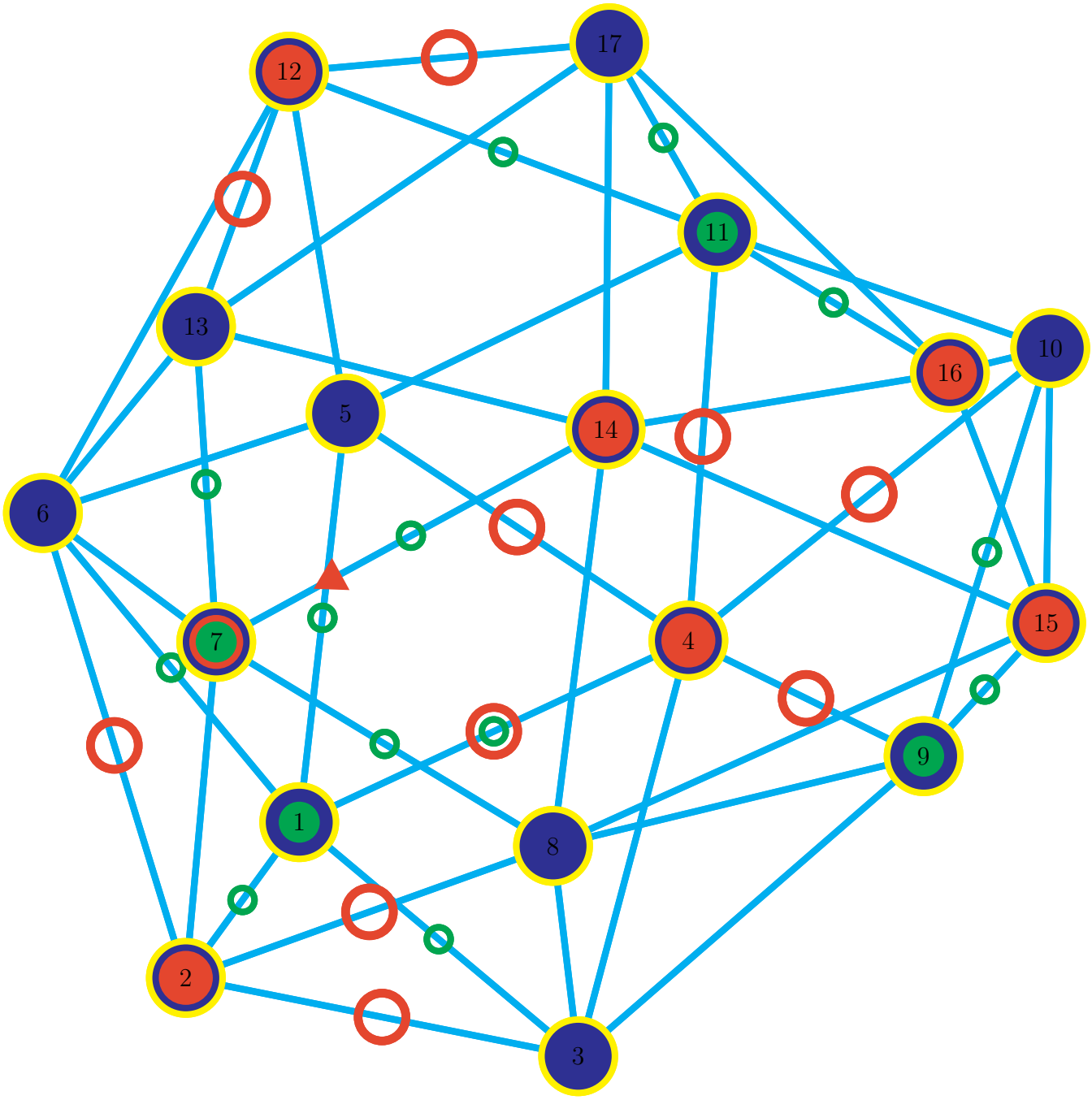

Figure 48: Instructions: 139: place edge 14→7 Red ArrowR

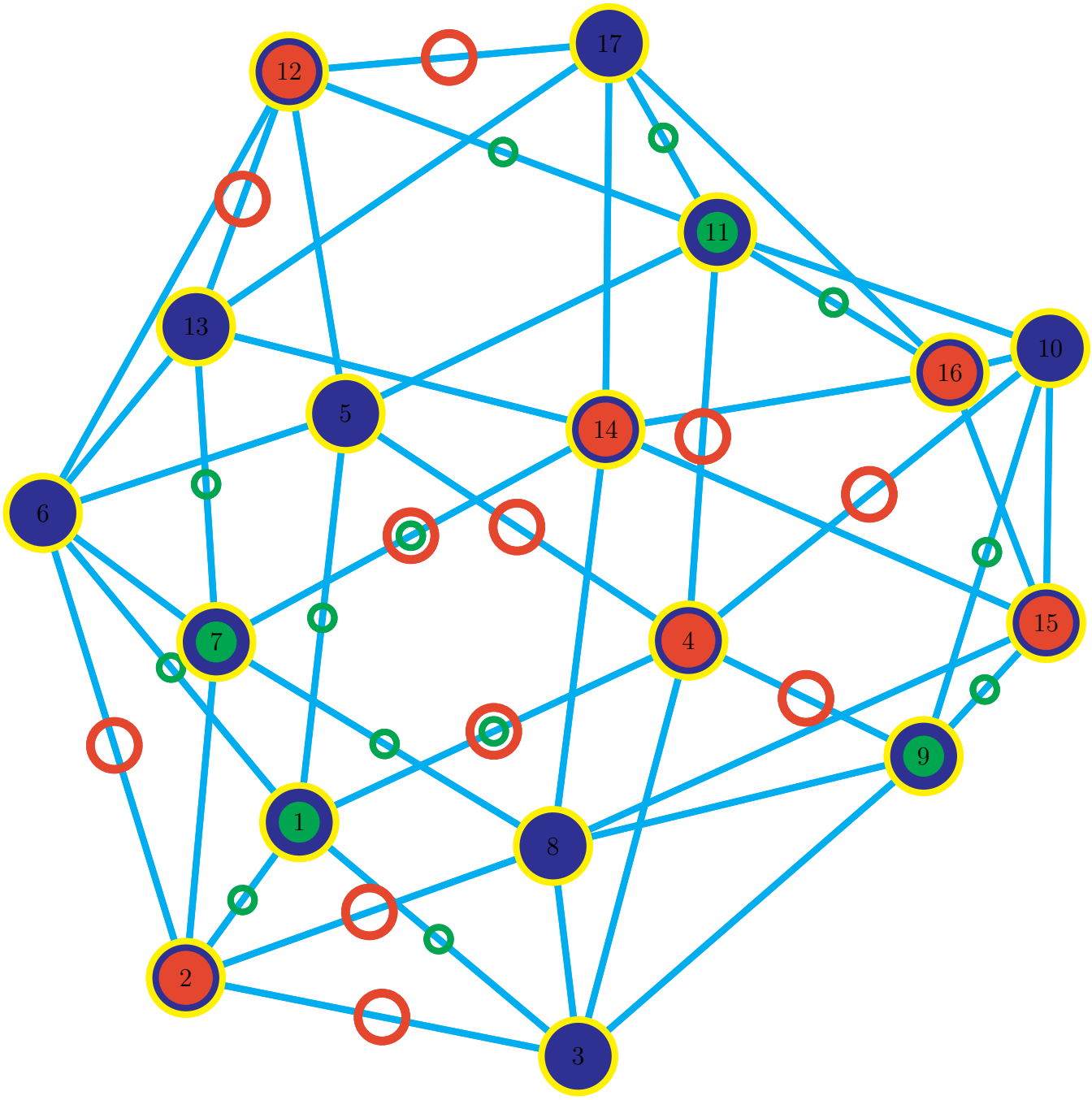

Figure 49: Instructions: 142: color edge 14→7 Red 143: uncolor vertex 7 Red

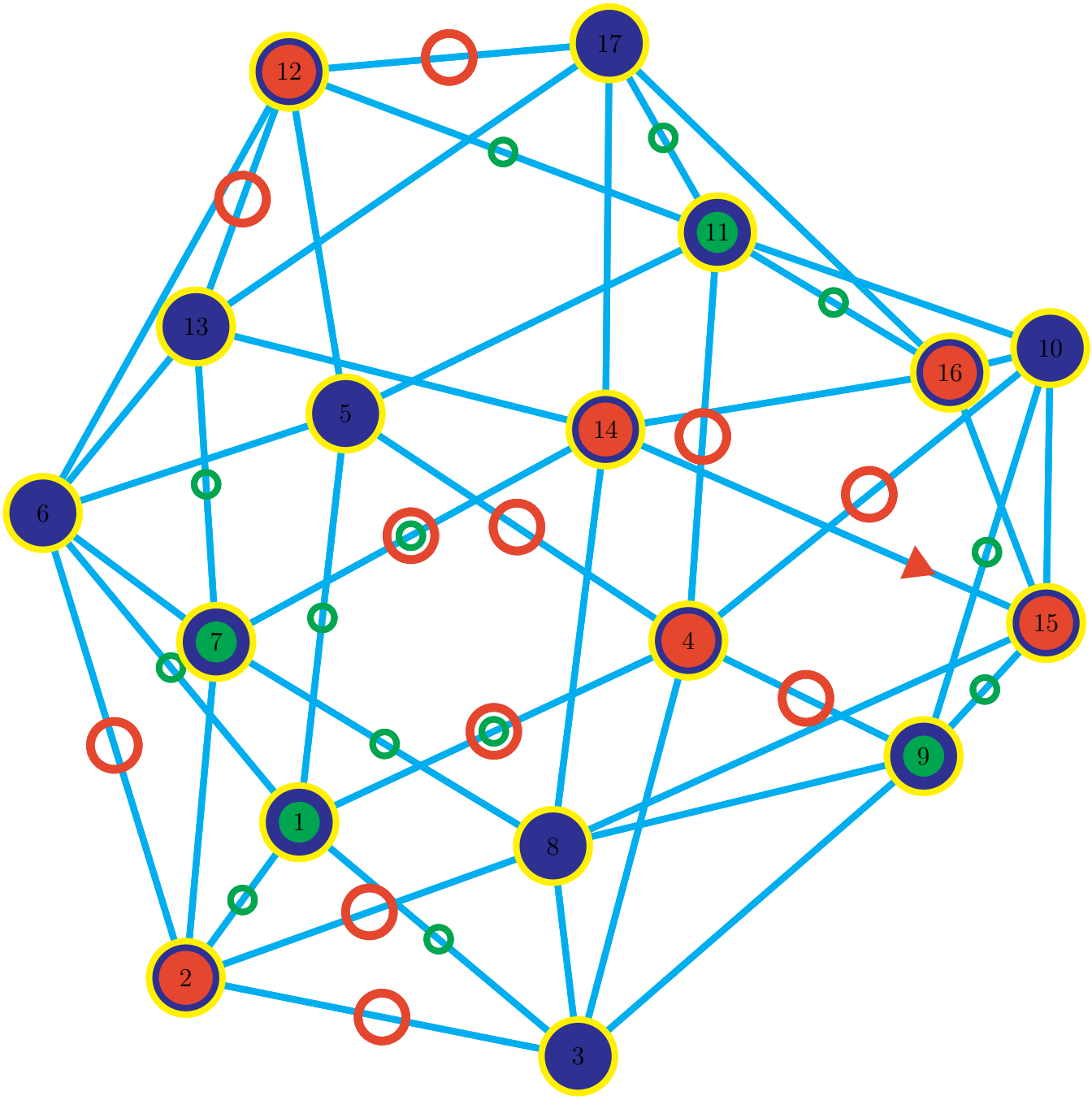

Figure 50: Instructions: 145: place edge 14→15 Red ArrowR

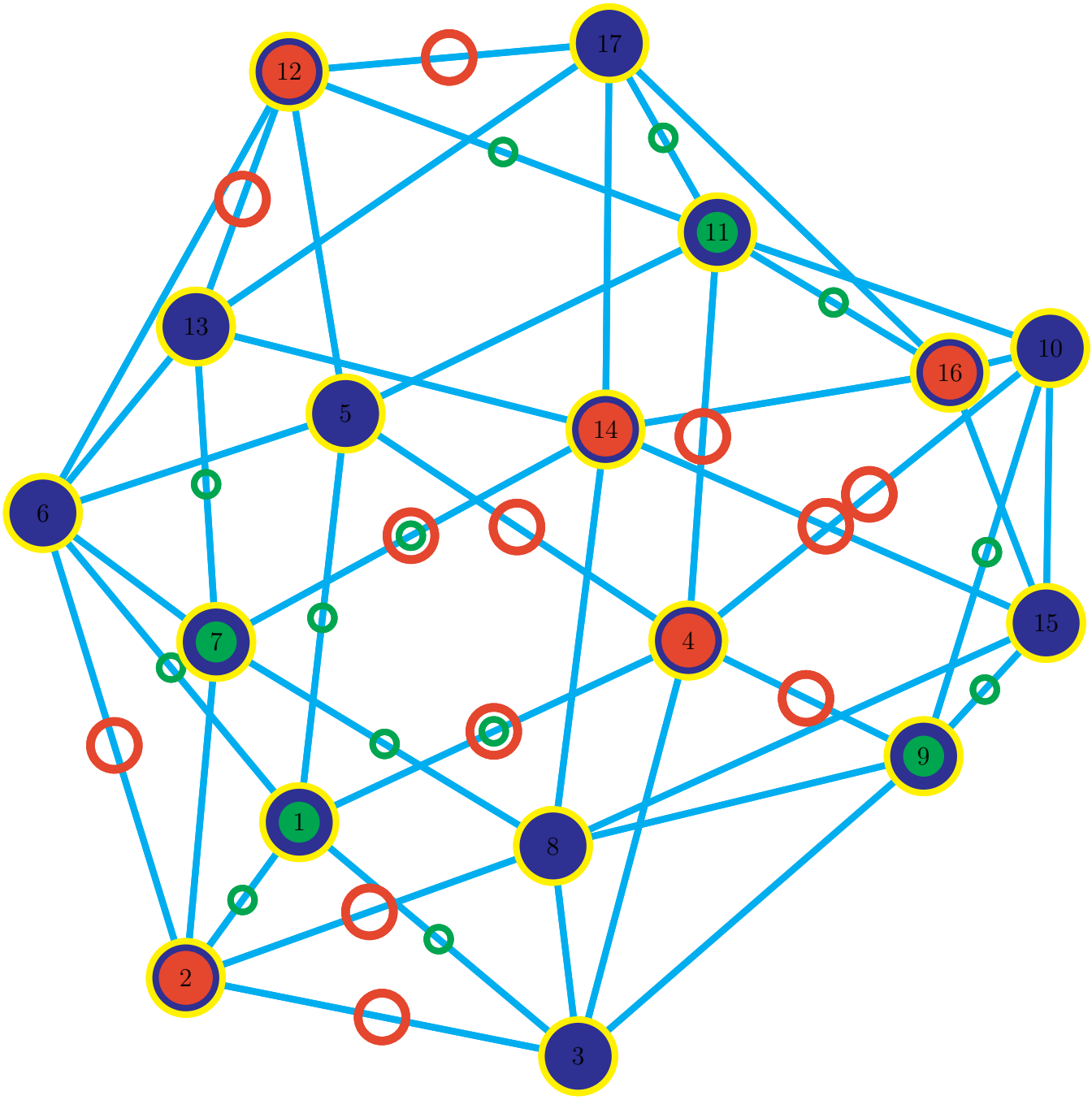

Figure 51: Instructions: 148: color edge 14→15 Red 149: uncolor vertex 15 Red

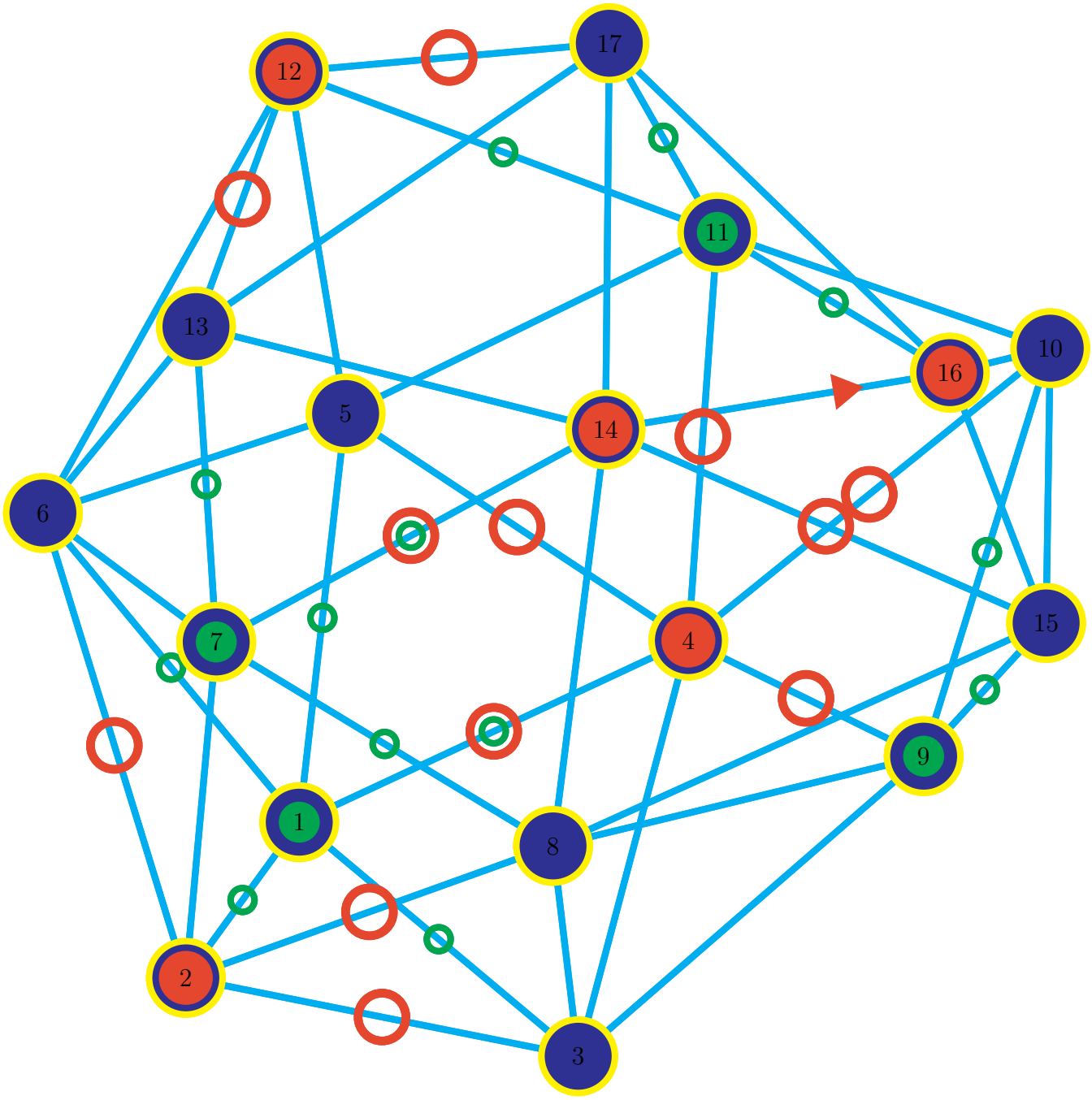

Figure 52: Instructions: 151: place edge 14→16 Red ArrowR

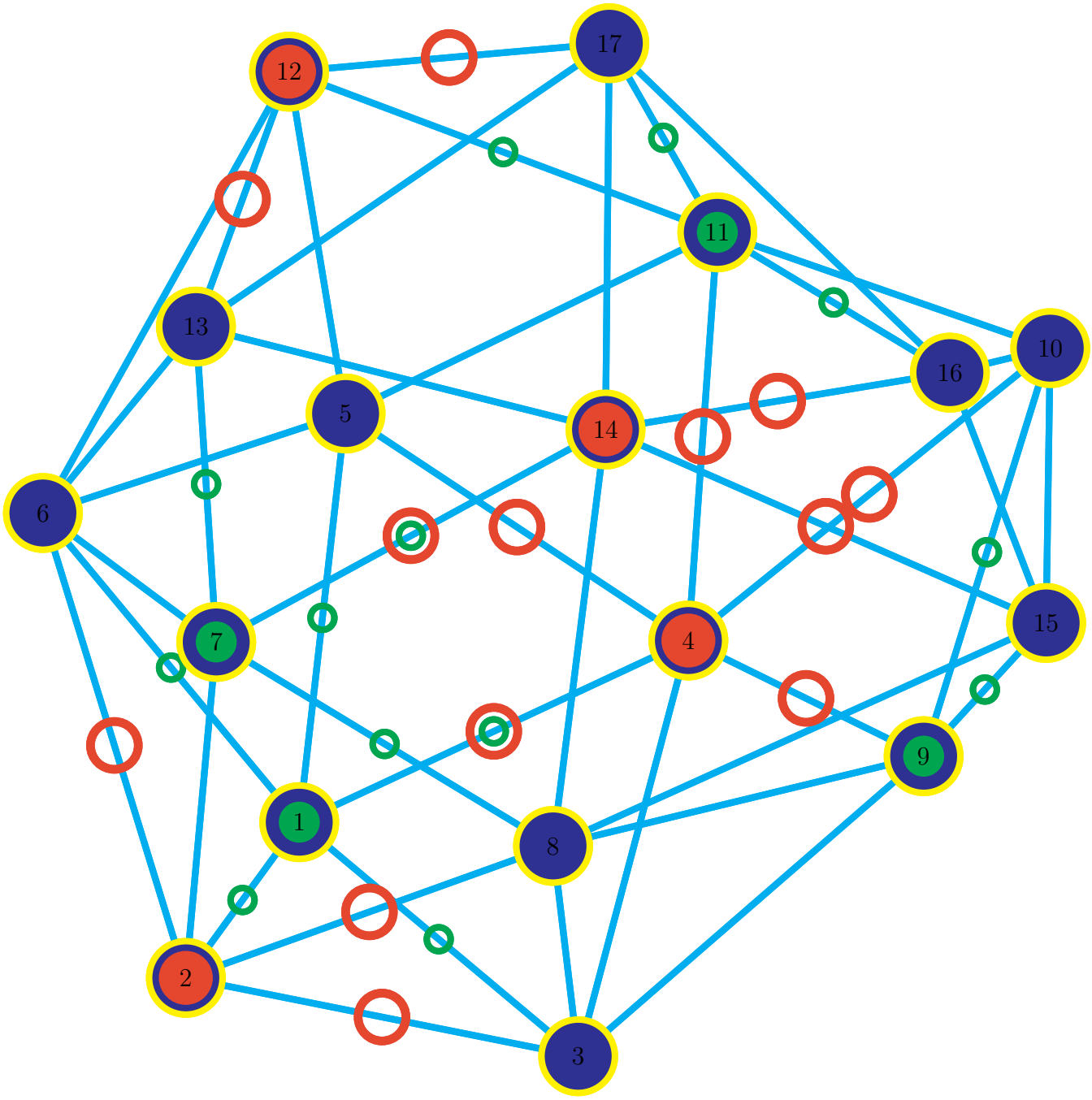

Figure 53: Instructions: 154: color edge 14→16 Red 155: uncolor vertex 16 Red

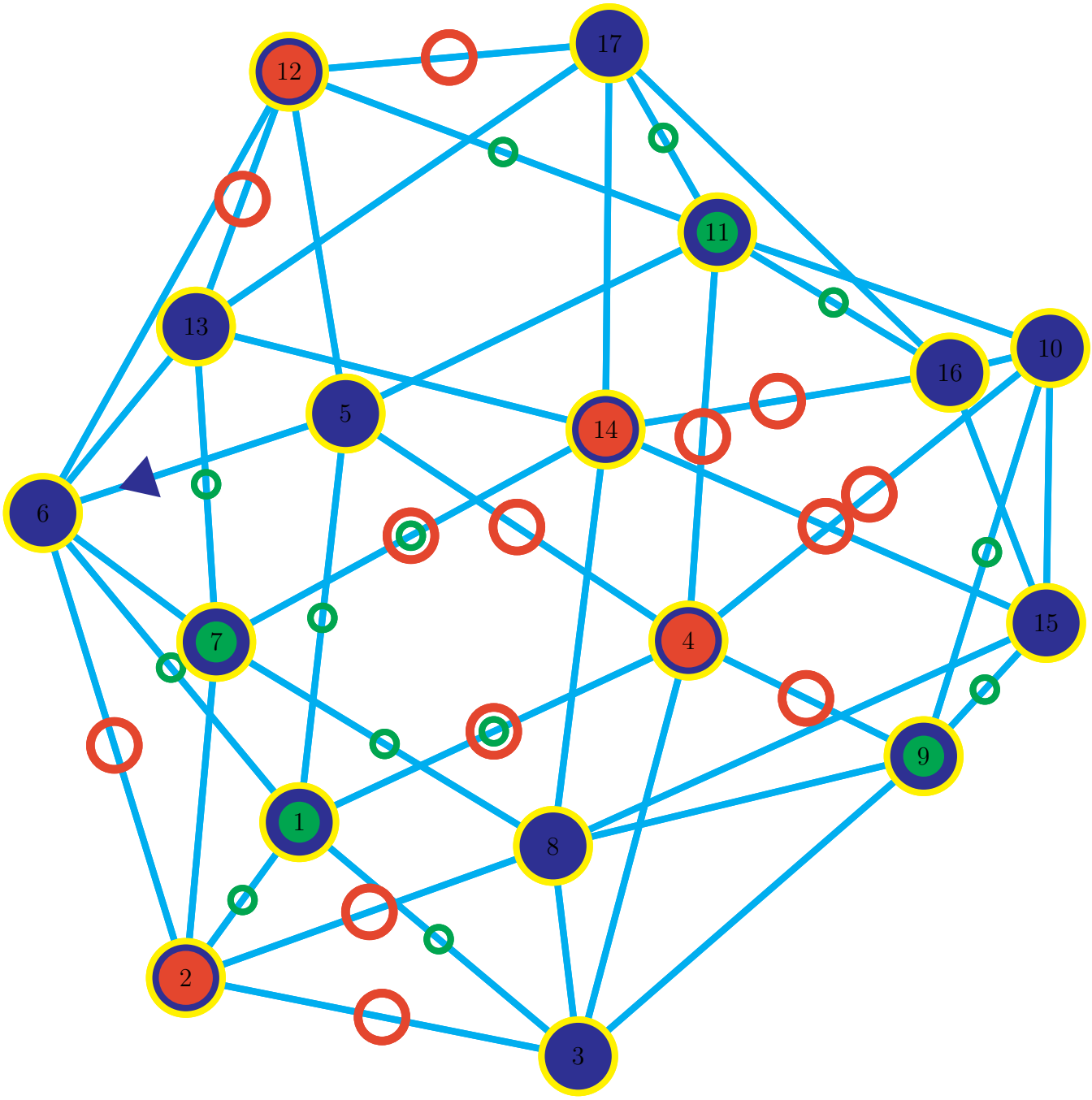

Figure 54: Instructions: 157: place edge 5→6 Blue ArrowR

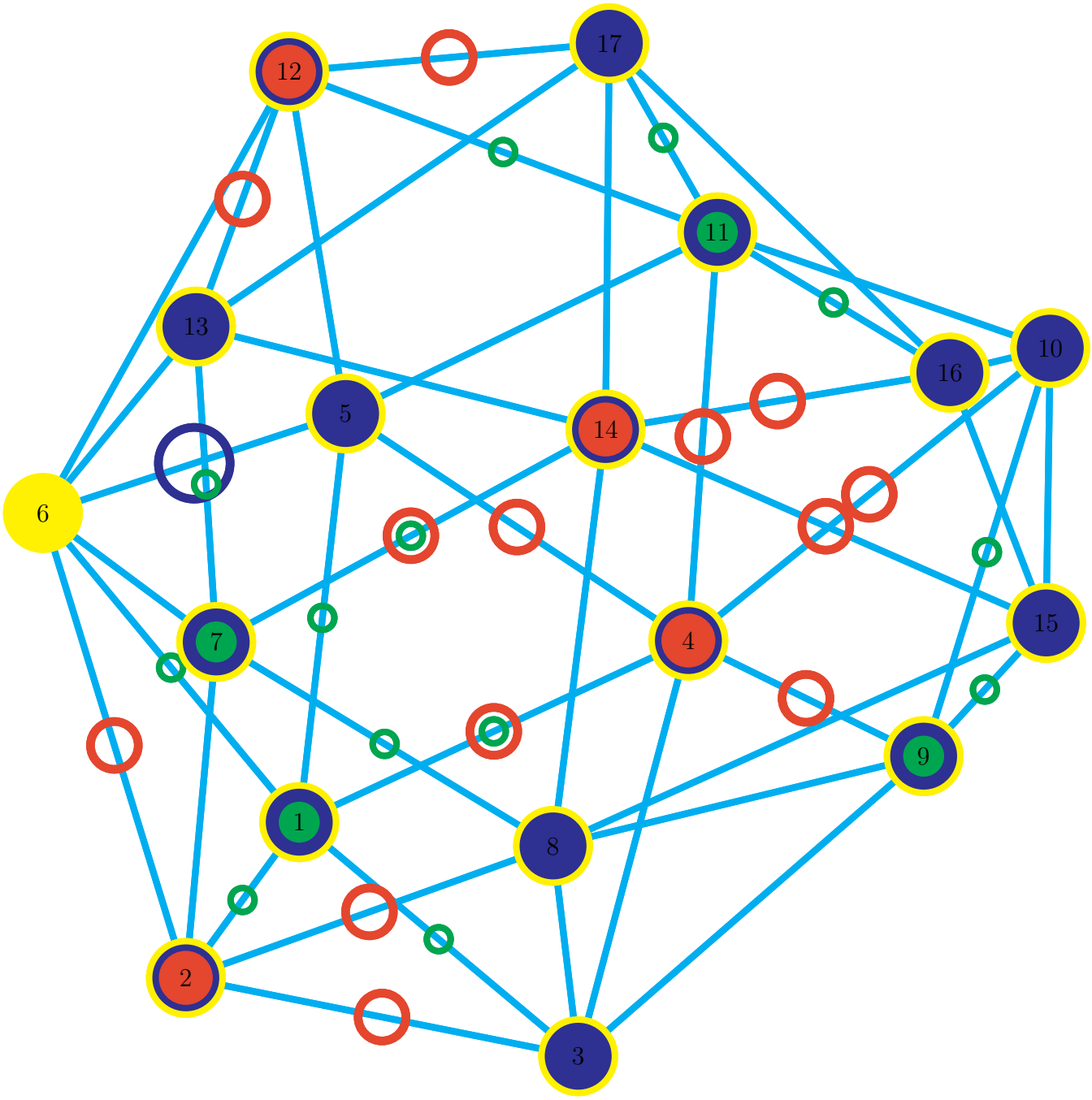

Figure 55: Instructions: 160: color edge 5→6 Blue 161: uncolor vertex 6 Blue

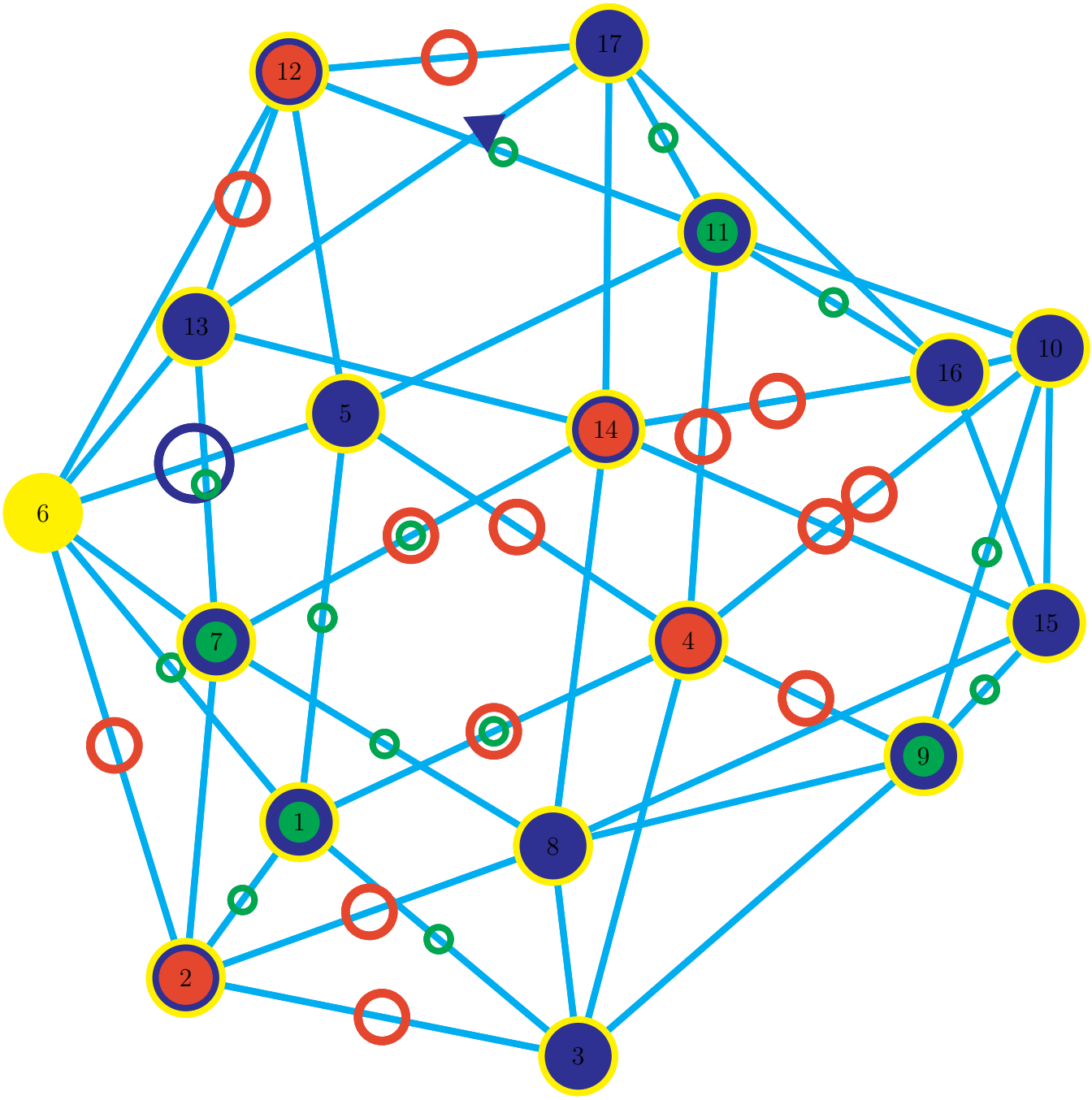

Figure 56: Instructions: 163: place edge 13→17 Blue ArrowR

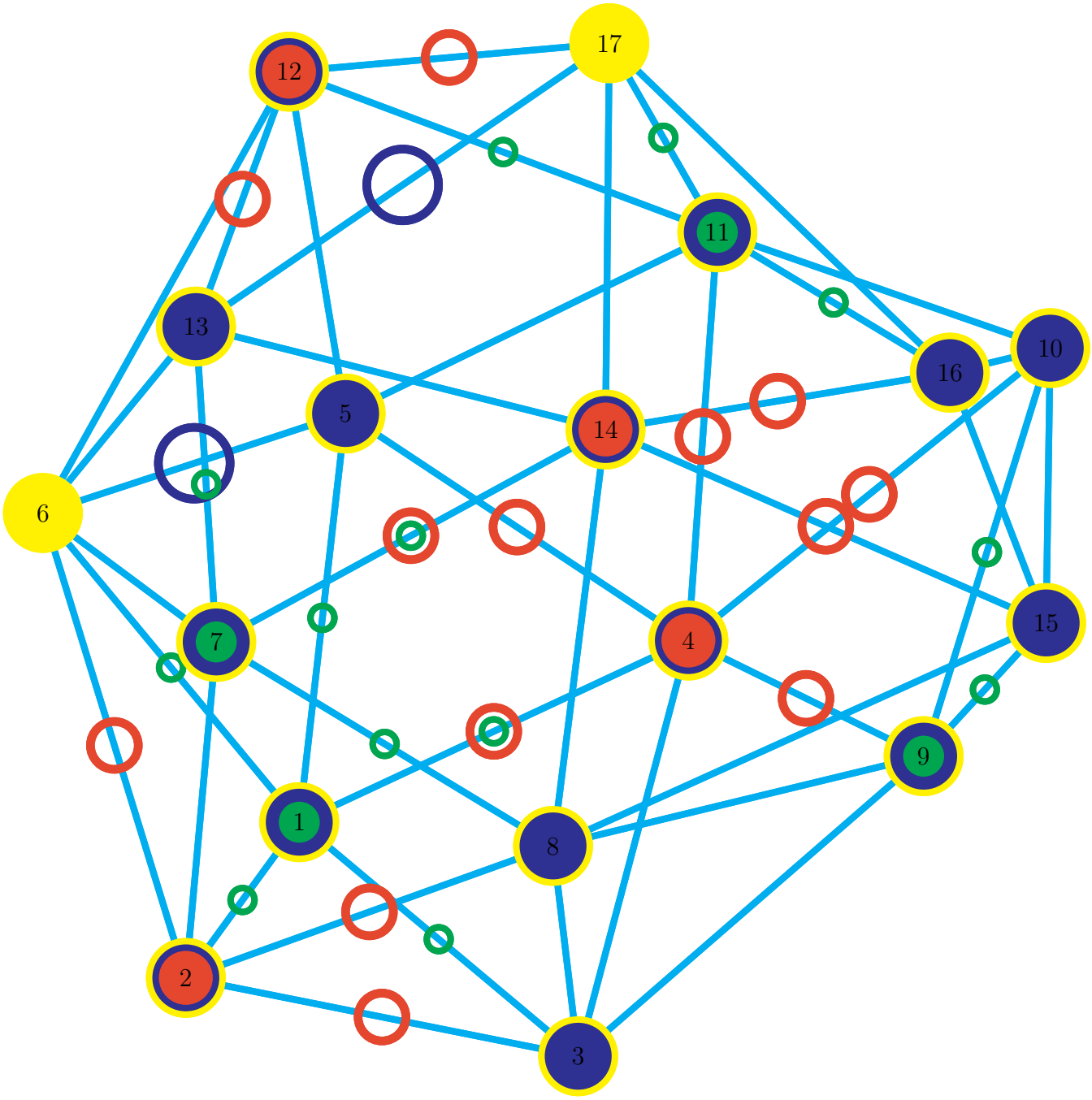

Figure 57: Instructions: 166: color edge 13→17 Blue 167: uncolor vertex 17 Blue

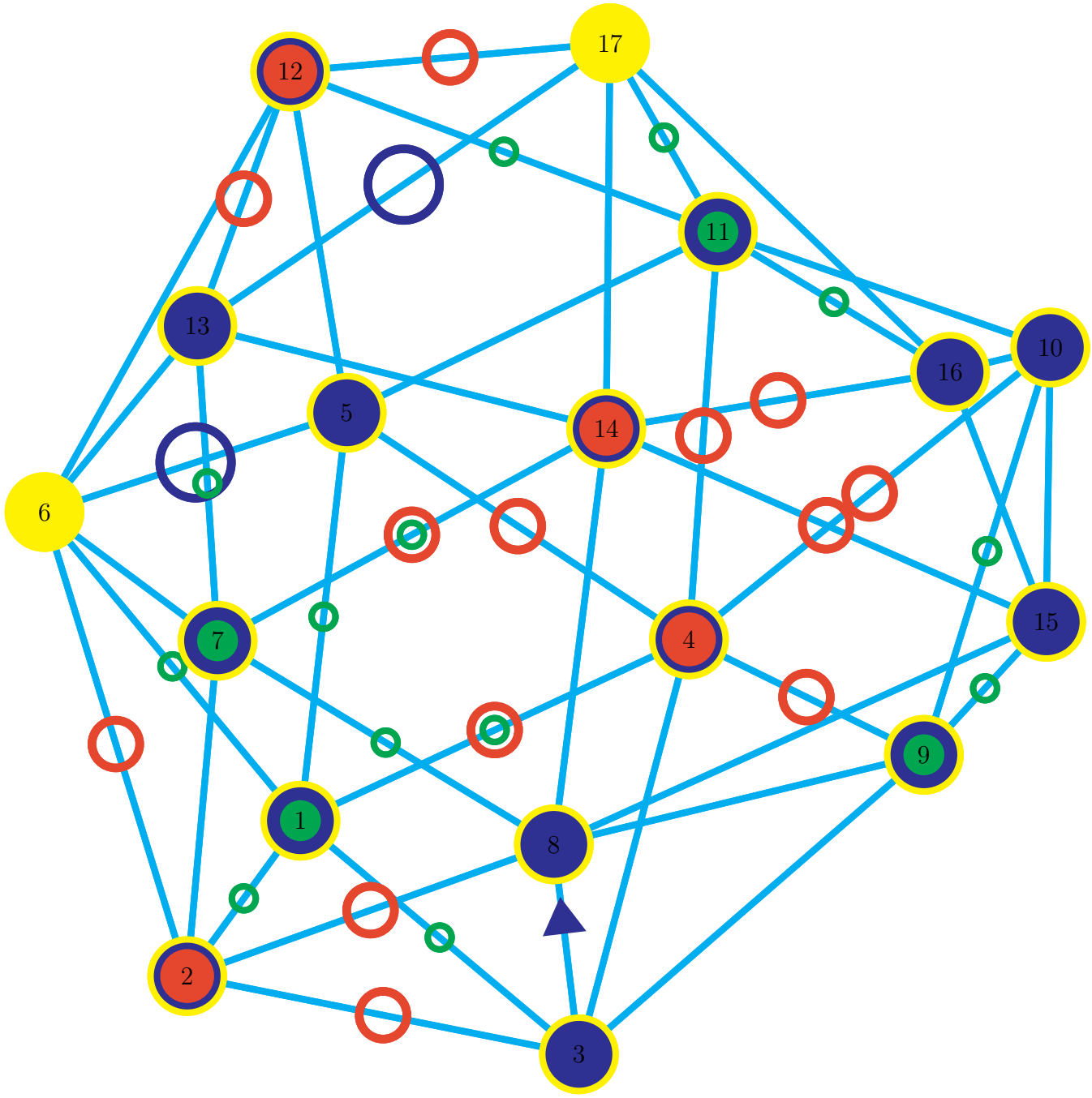

Figure 58: Instructions: 169: place edge 3→8 Blue ArrowR

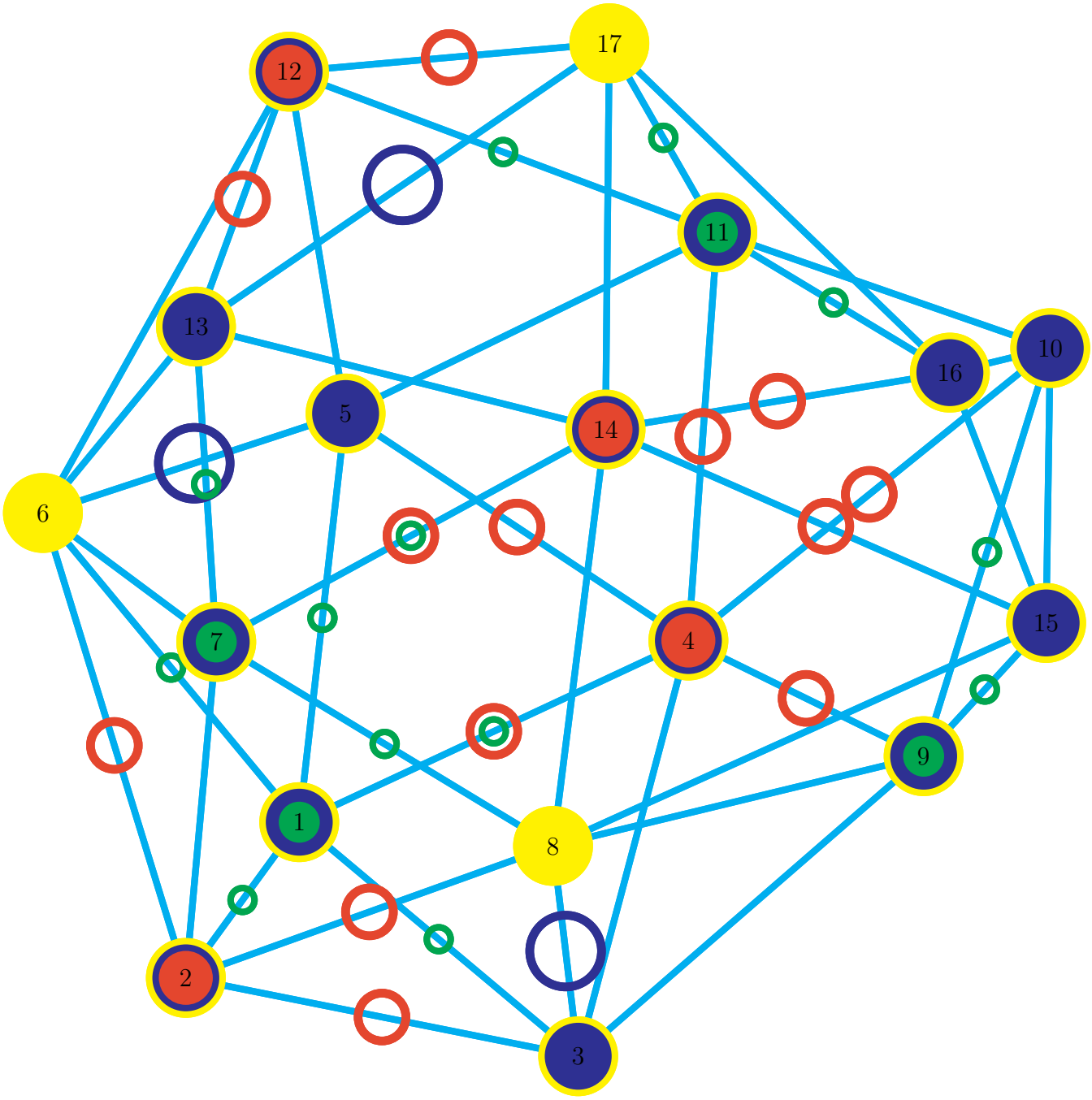

Figure 59: Instructions: 172: color edge 3→8 Blue 173: uncolor vertex 8 Blue

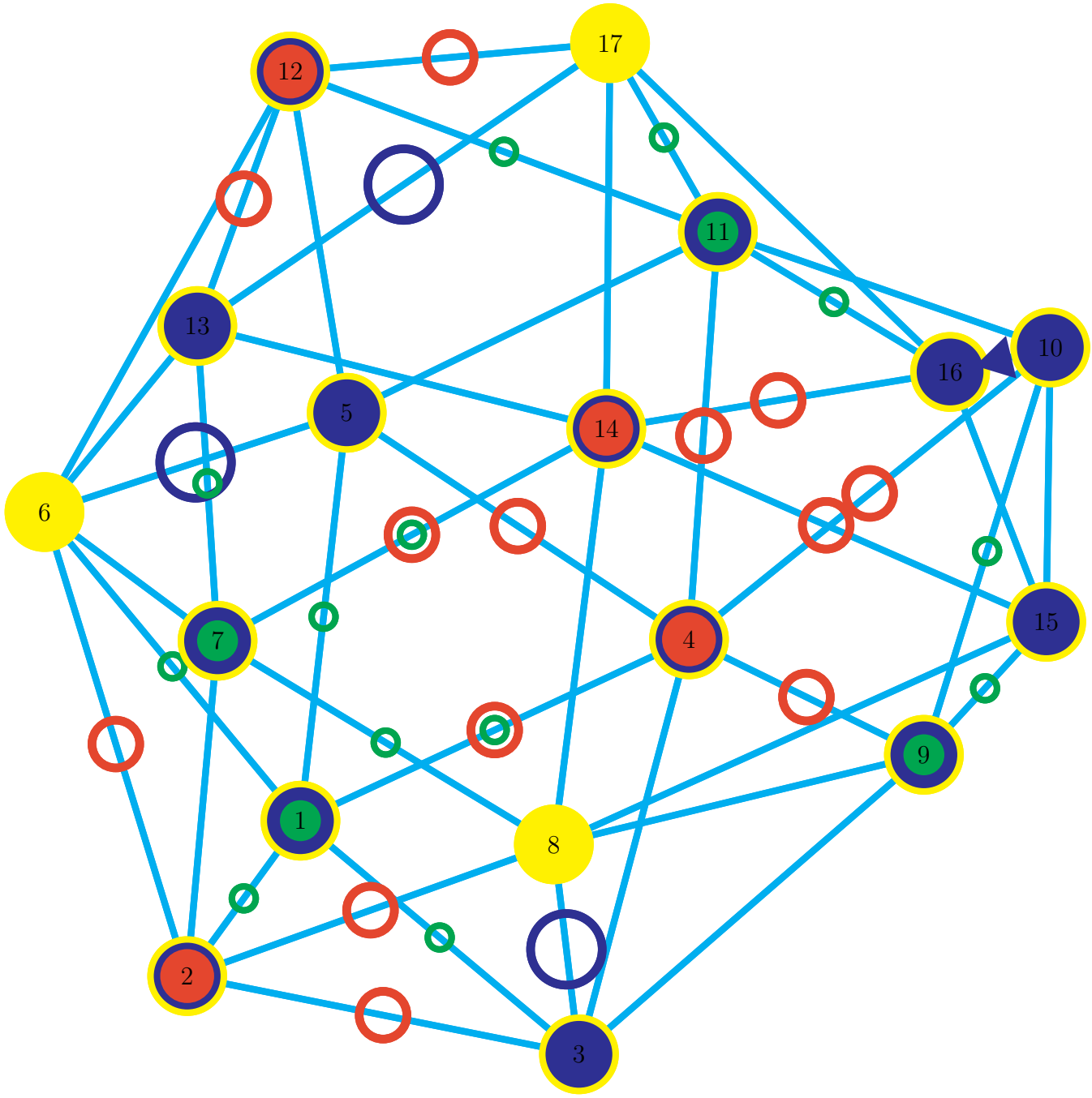

Figure 60: Instructions: 175: place edge 10→16 Blue ArrowR

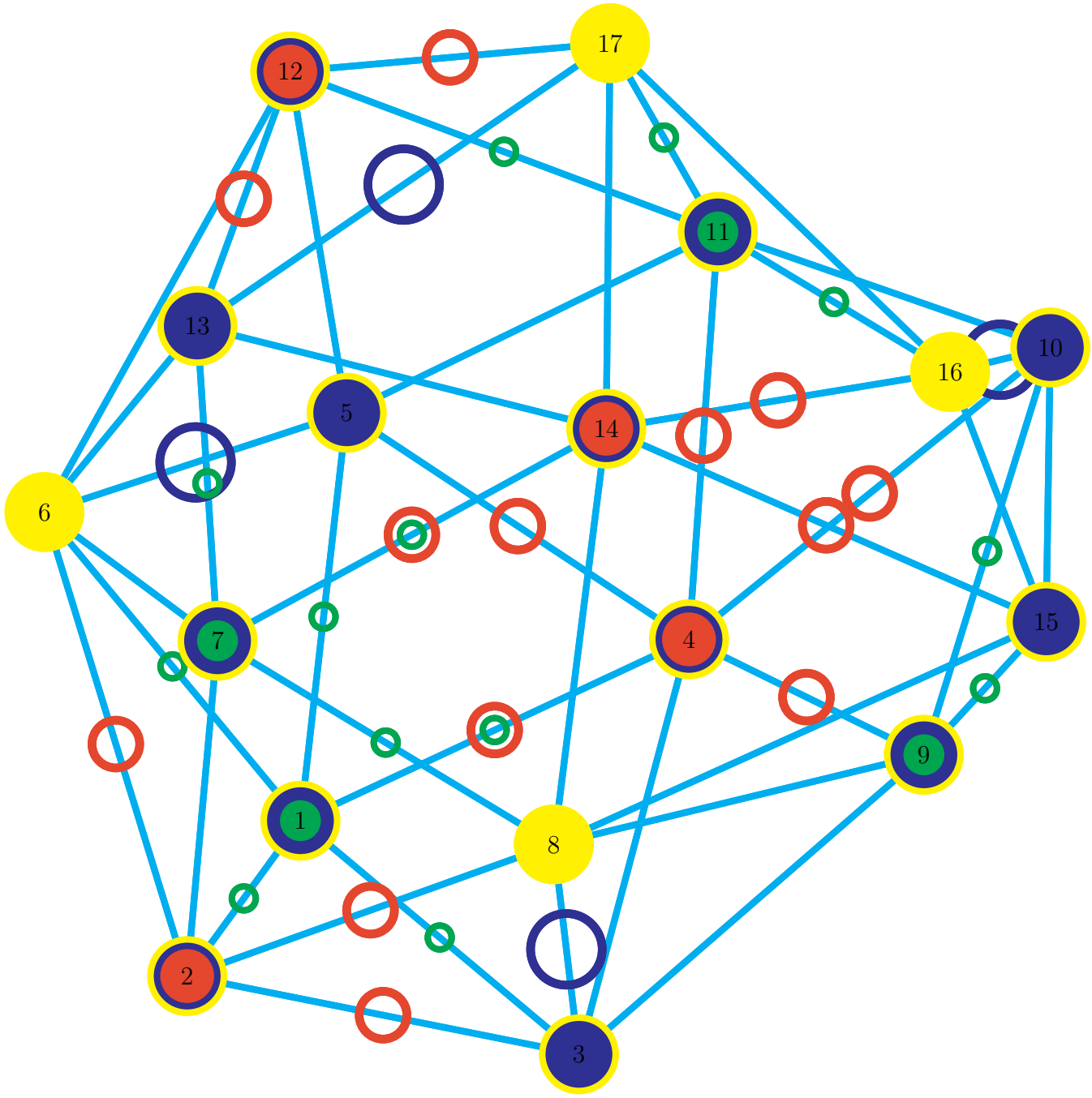

Figure 61: Instructions: 178: color edge 10→16 Blue 179: uncolor vertex 16 Blue

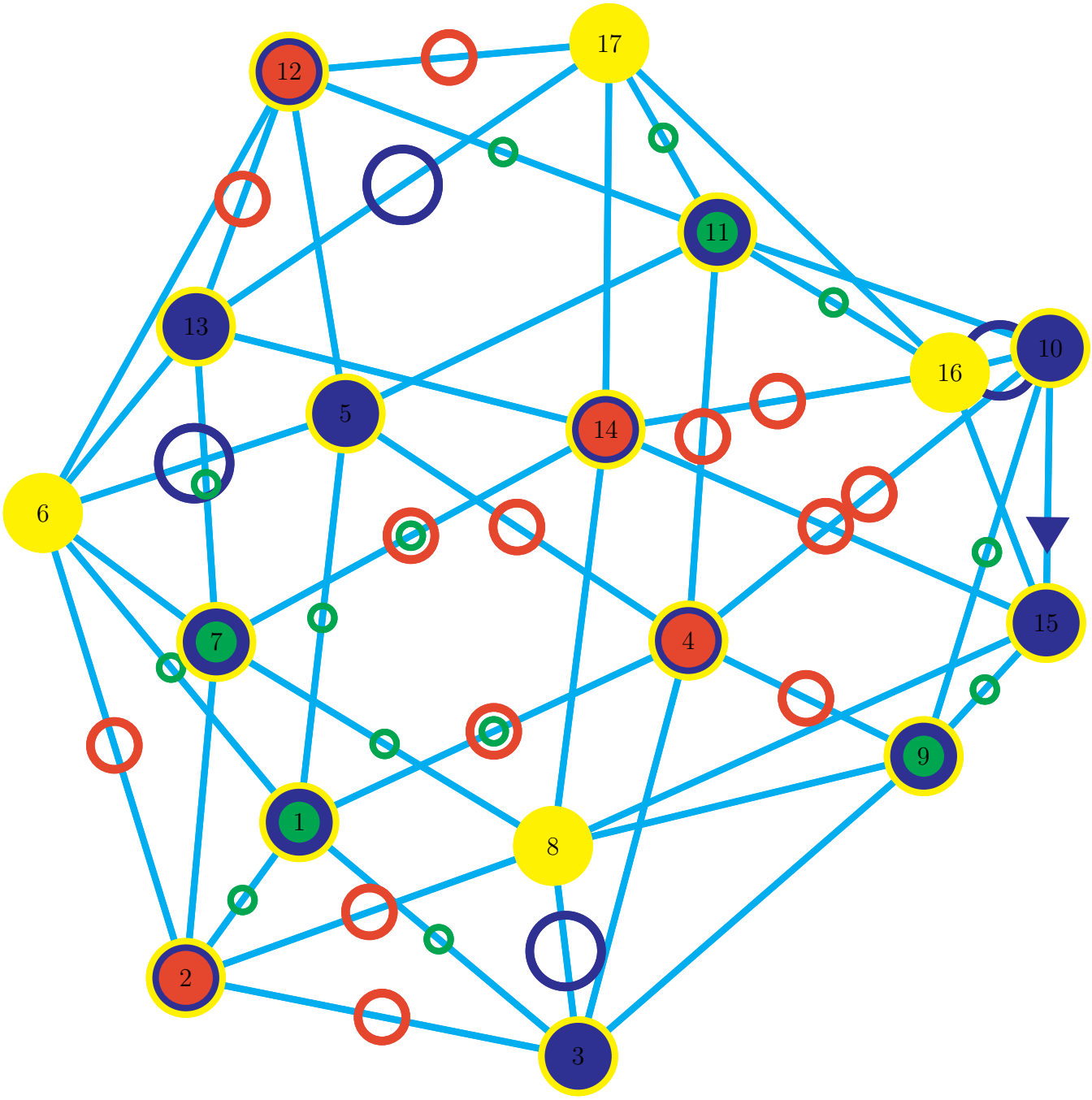

Figure 62: Instructions: 181: place edge 10→15 Blue ArrowR

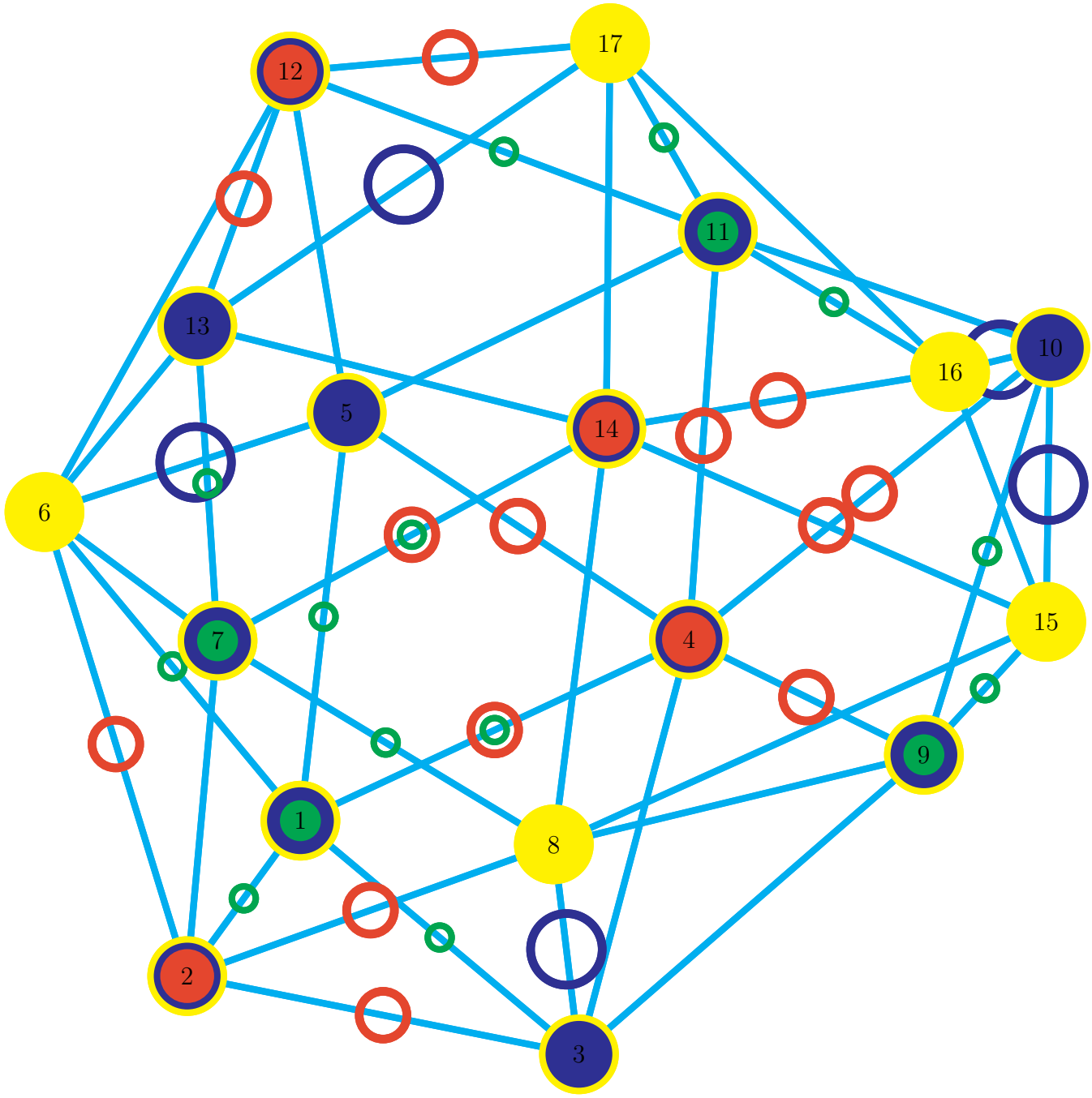

Figure 63: Instructions: 184: color edge 10→15 Blue 185: uncolor vertex 15 Blue

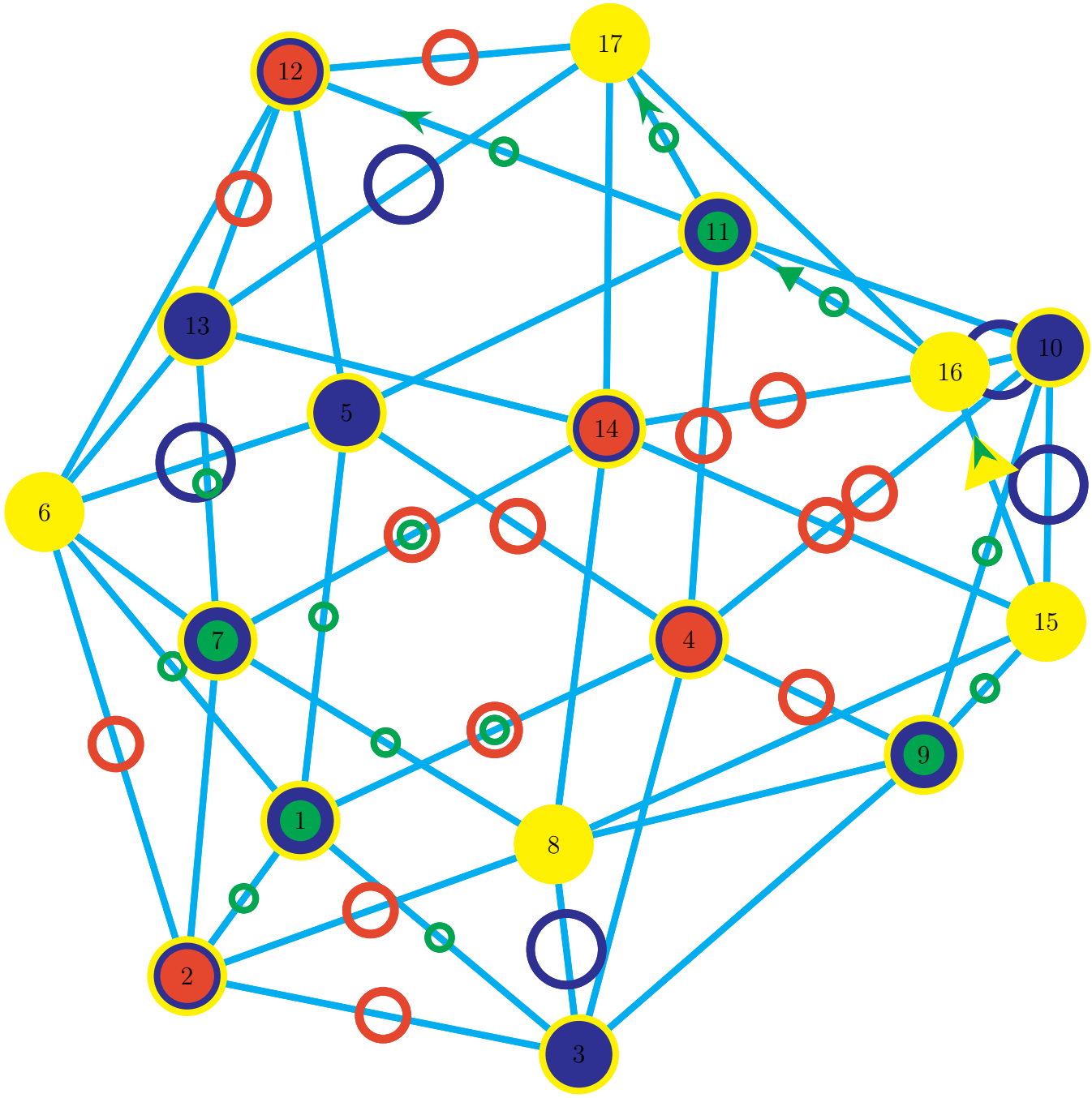

Figure 64: Instructions: 187: place edge 15→16 Yellow ArrowR 188: place edge 15→16 Green ArrowI 189: place edge 16→11 Green ArrowR 190: place edge 11→17 Green ArrowI 191: place edge 11→12 Green ArrowI

This is the first alternating chain of length greater than 1. It is also branched. Insertion arrows are used for the first time. They are distinguished by having pointed tails, whereas the deletion arrows have flat tails. There are two arrowheads pointing from vertex 15 to vertex 16. A yellow deletion arrow and a green insertion arrow. They indicate the deletion of a yellow checker on vertex 16 and the insertion of a green checker. There are also a deletion arrow pointing to vertex 11, and insertion arrows pointing to vertices 12 abd 17.

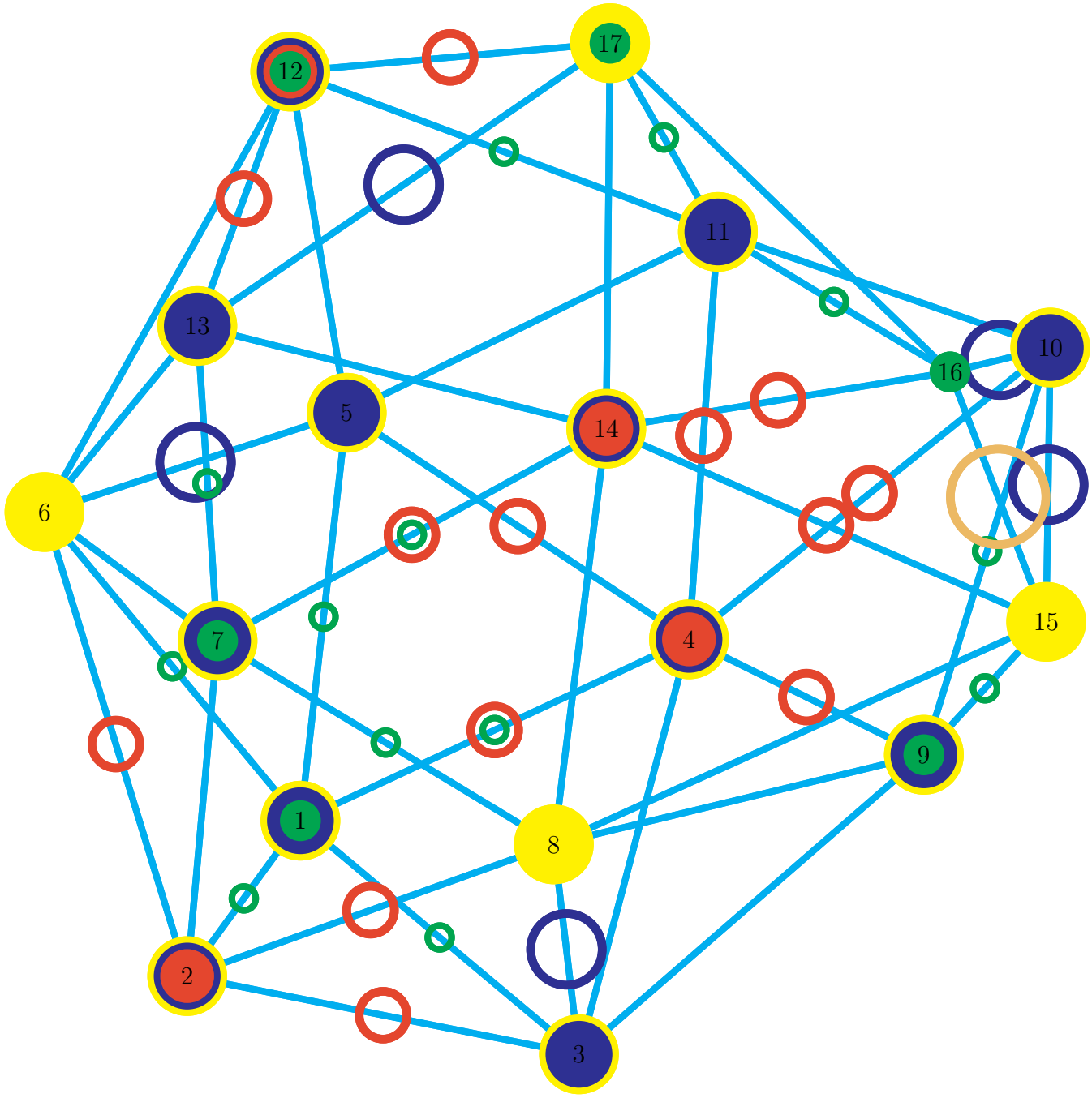

Figure 65: Instructions: 198: color edge 15→16 Yellow 199: uncolor vertex 16 Yellow 200: color vertex 16 Green 201: uncolor vertex 11 Green 202: color vertex 17 Green 203: color vertex 12 Green

The checkers have been moved according to the arrows and a new Yellow circle has been placed on edge 15-16 indicating that there is no conflict on this edge wrt Yellow. Also Properties A and B obtain.

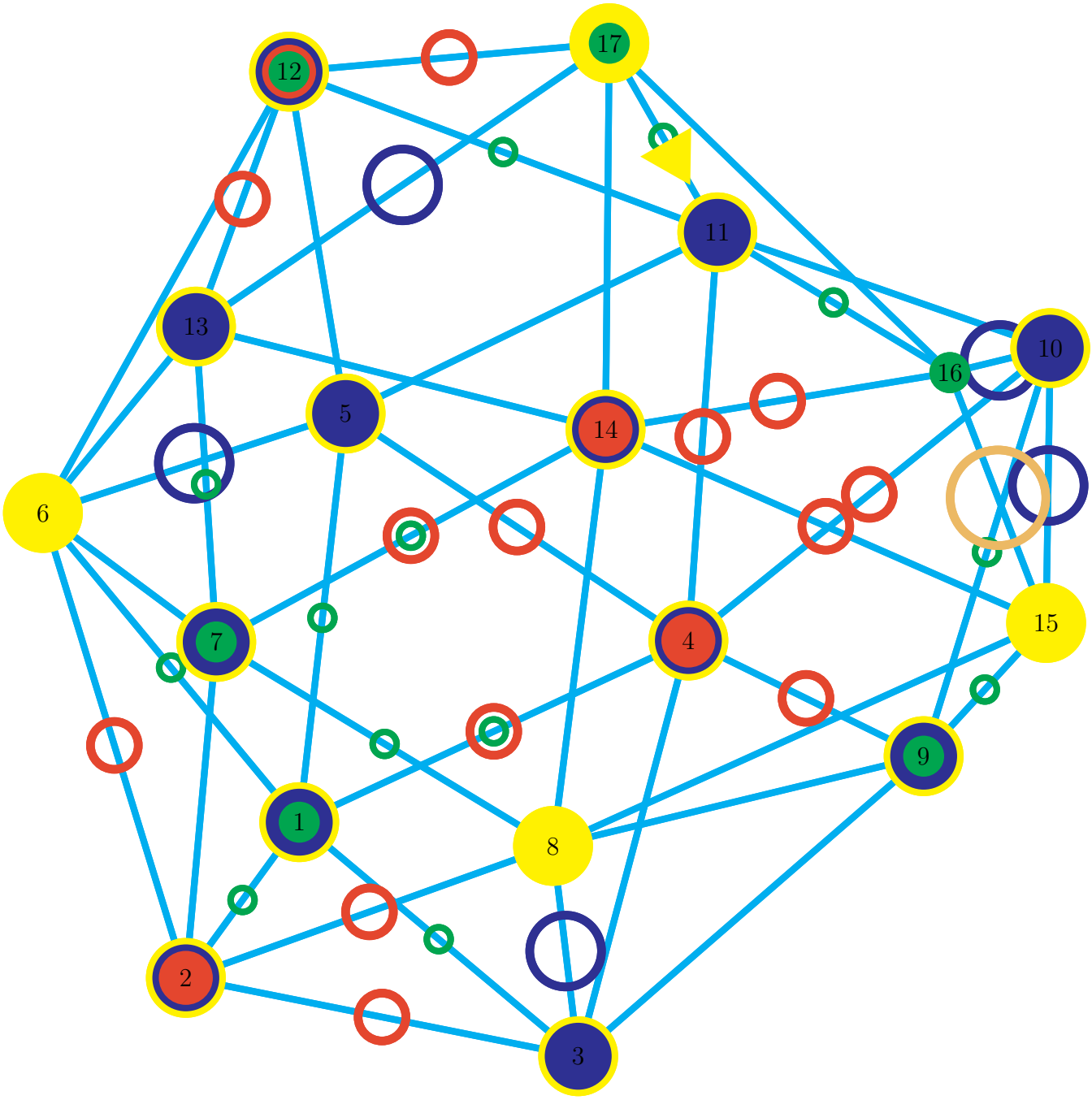

Figure 66: Instructions: 205: place edge 17→11 Yellow ArrowR

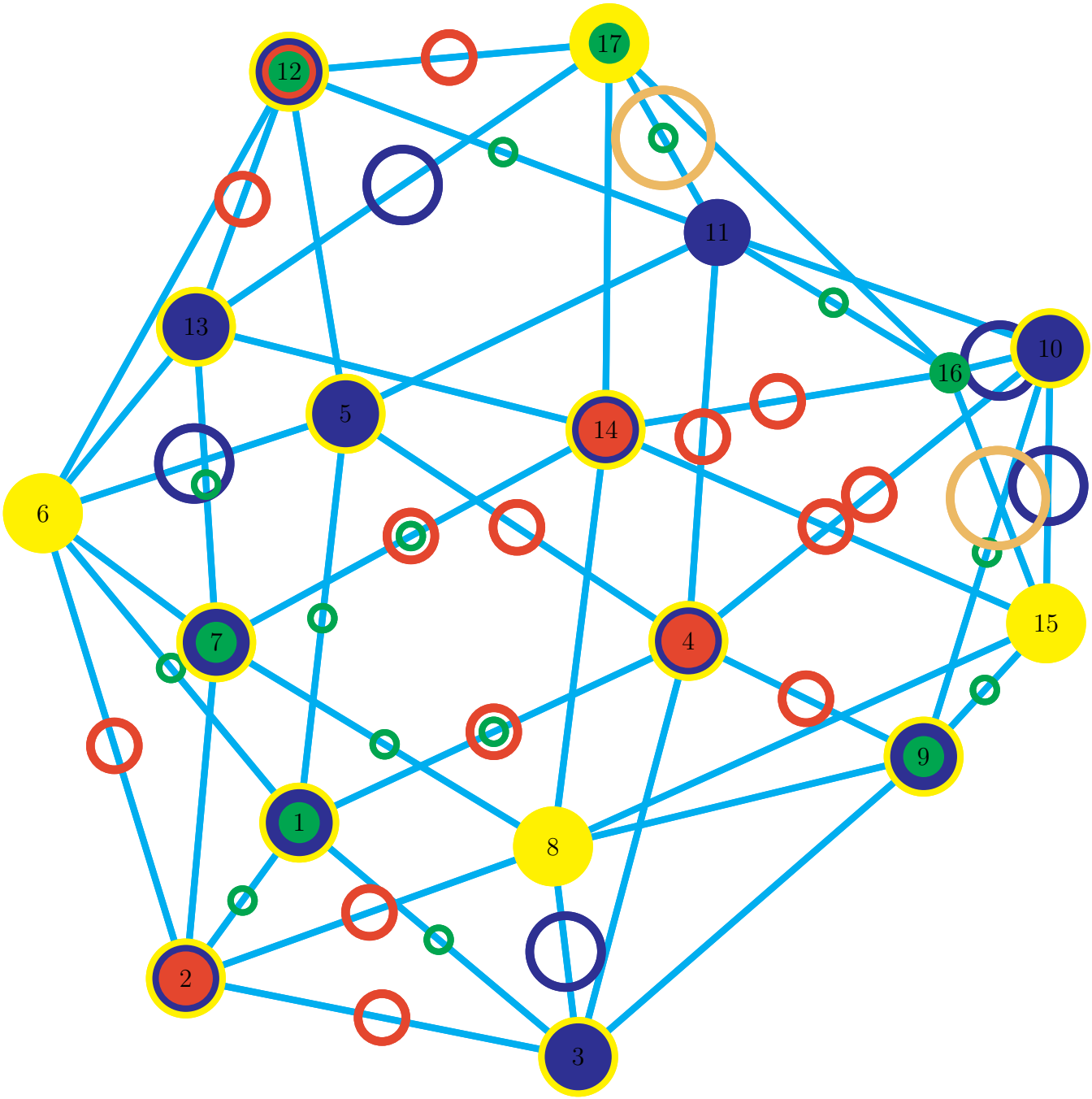

Figure 67: Instructions: 208: color edge 11→17 Yellow 209: uncolor vertex 11 Yellow

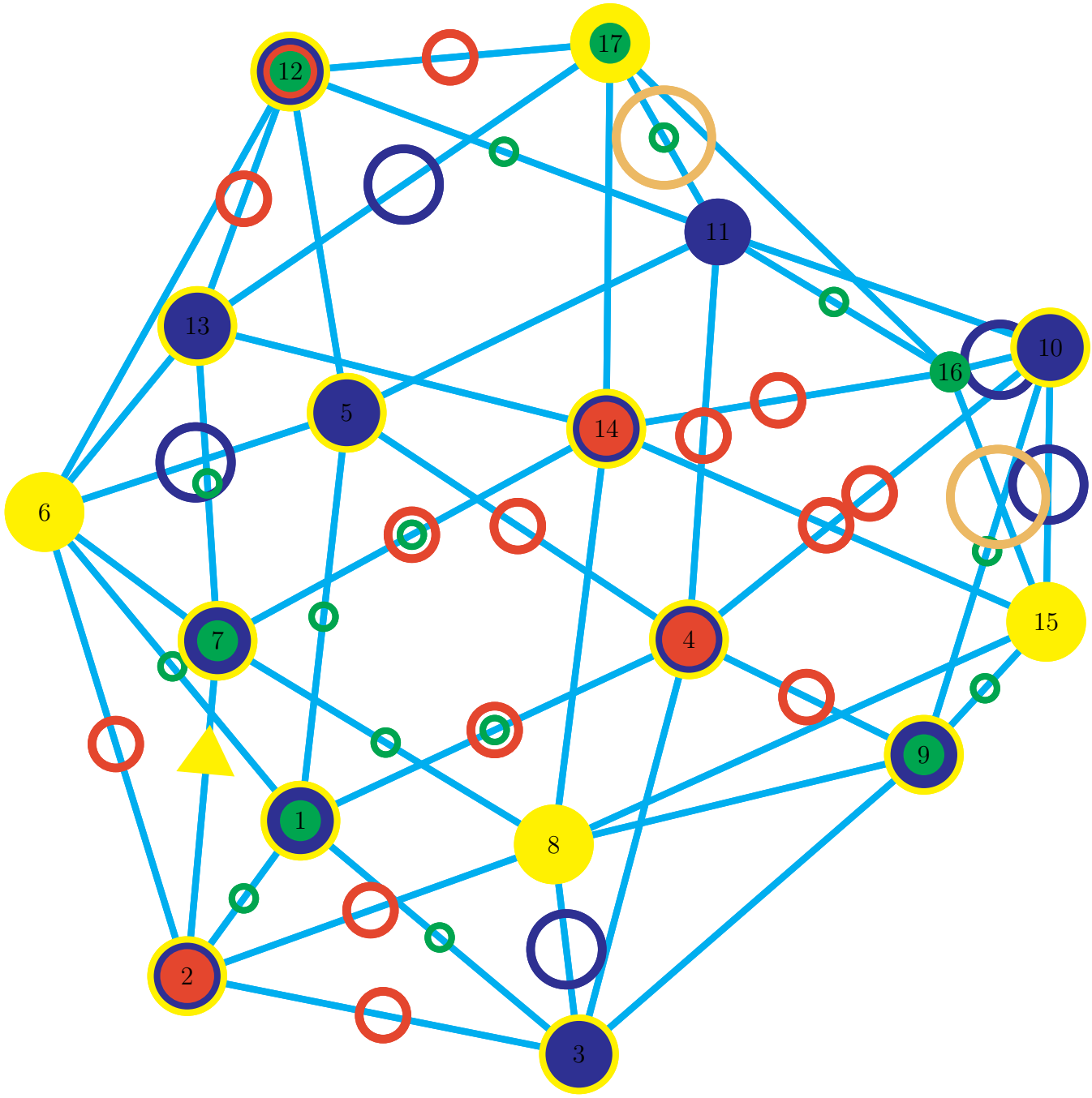

Figure 68: Instructions: 211: place edge 2→7 Yellow ArrowR

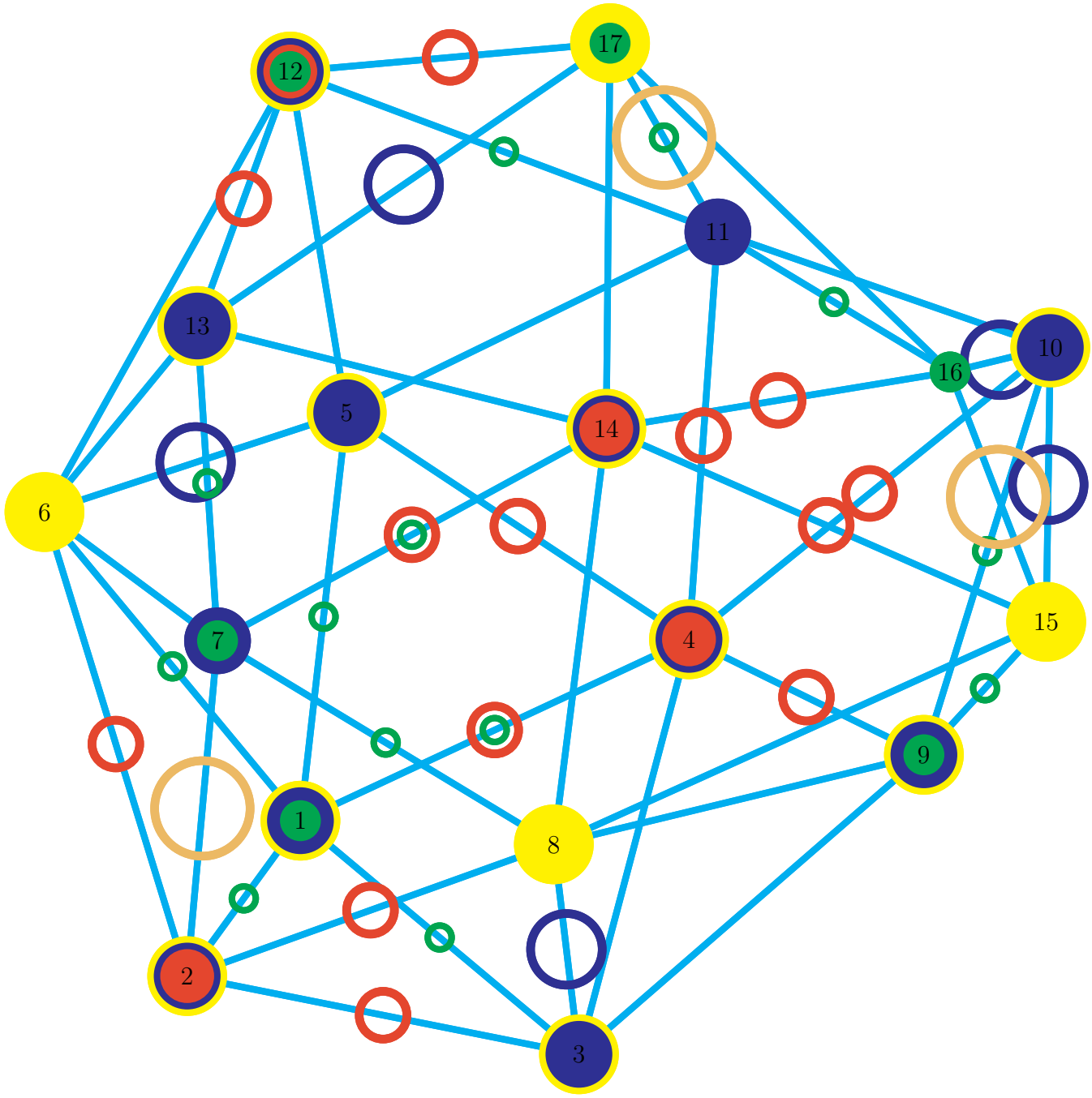

Figure 69: Instructions: 214: color edge 2→7 Yellow 215: uncolor vertex 7 Yellow

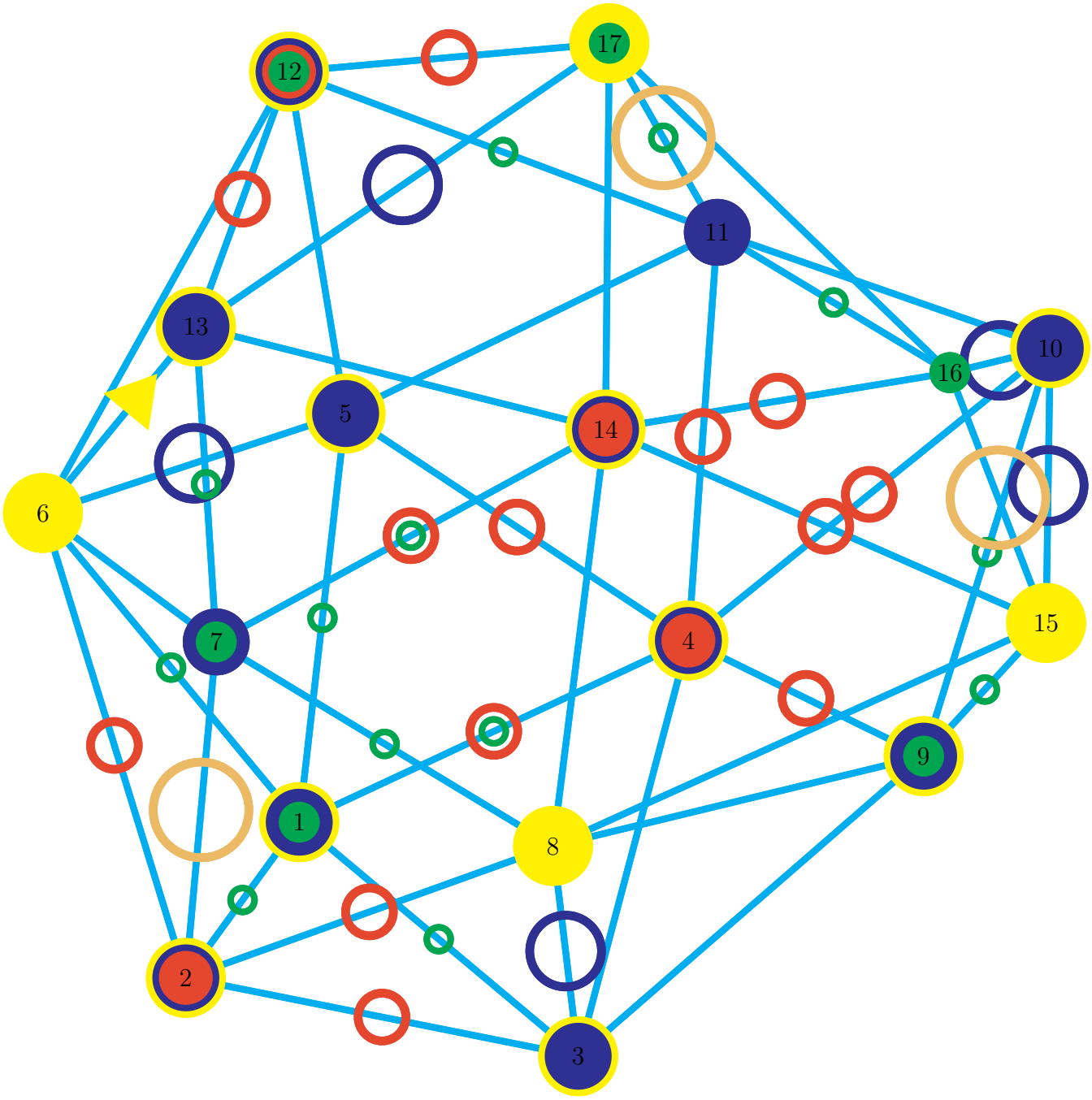

Figure 70: Instructions: 217: place edge 6→13 Yellow ArrowR

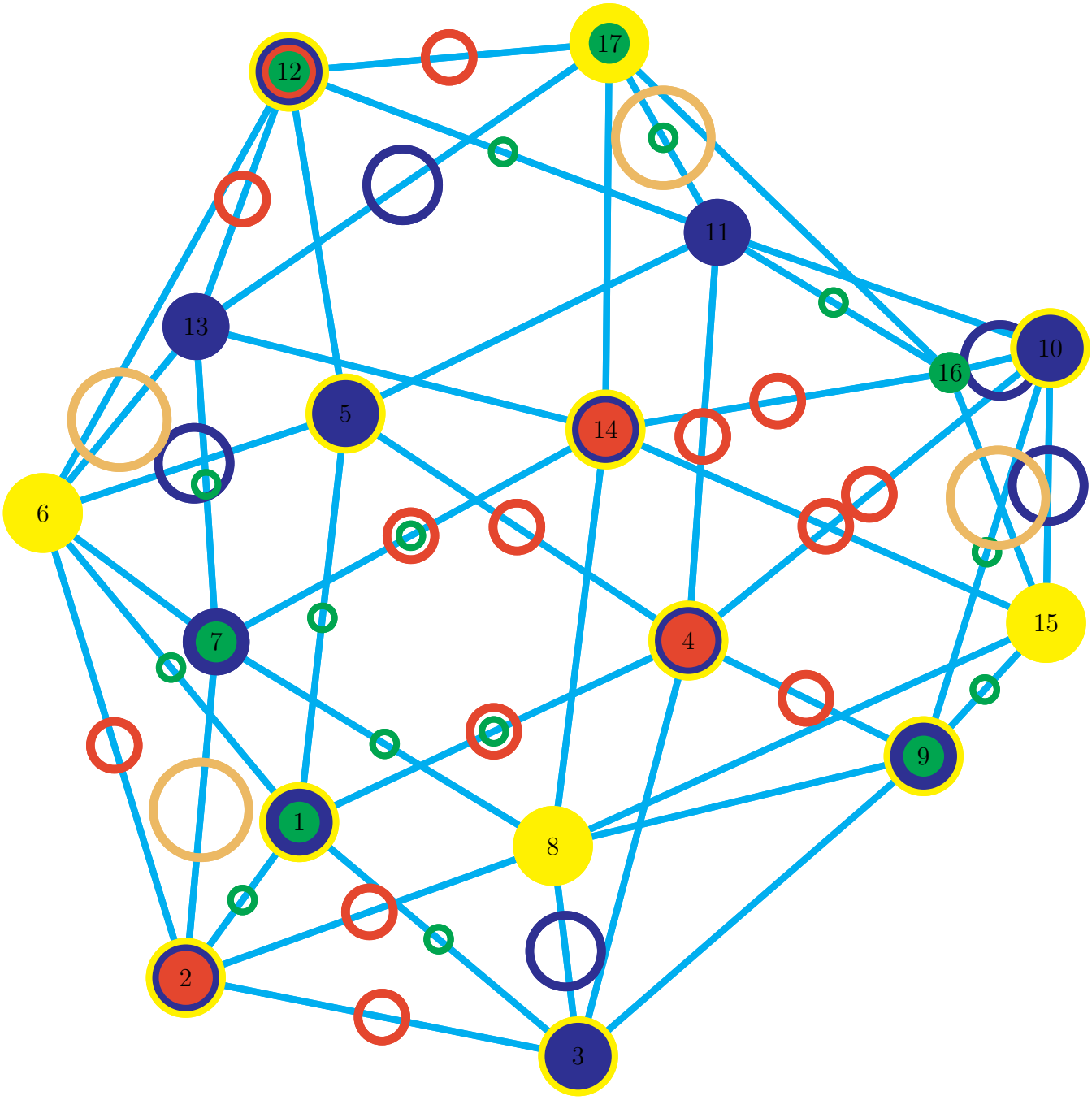

Figure 71: Instructions: 220: color edge 6→13 Yellow 221: uncolor vertex 13 Yellow

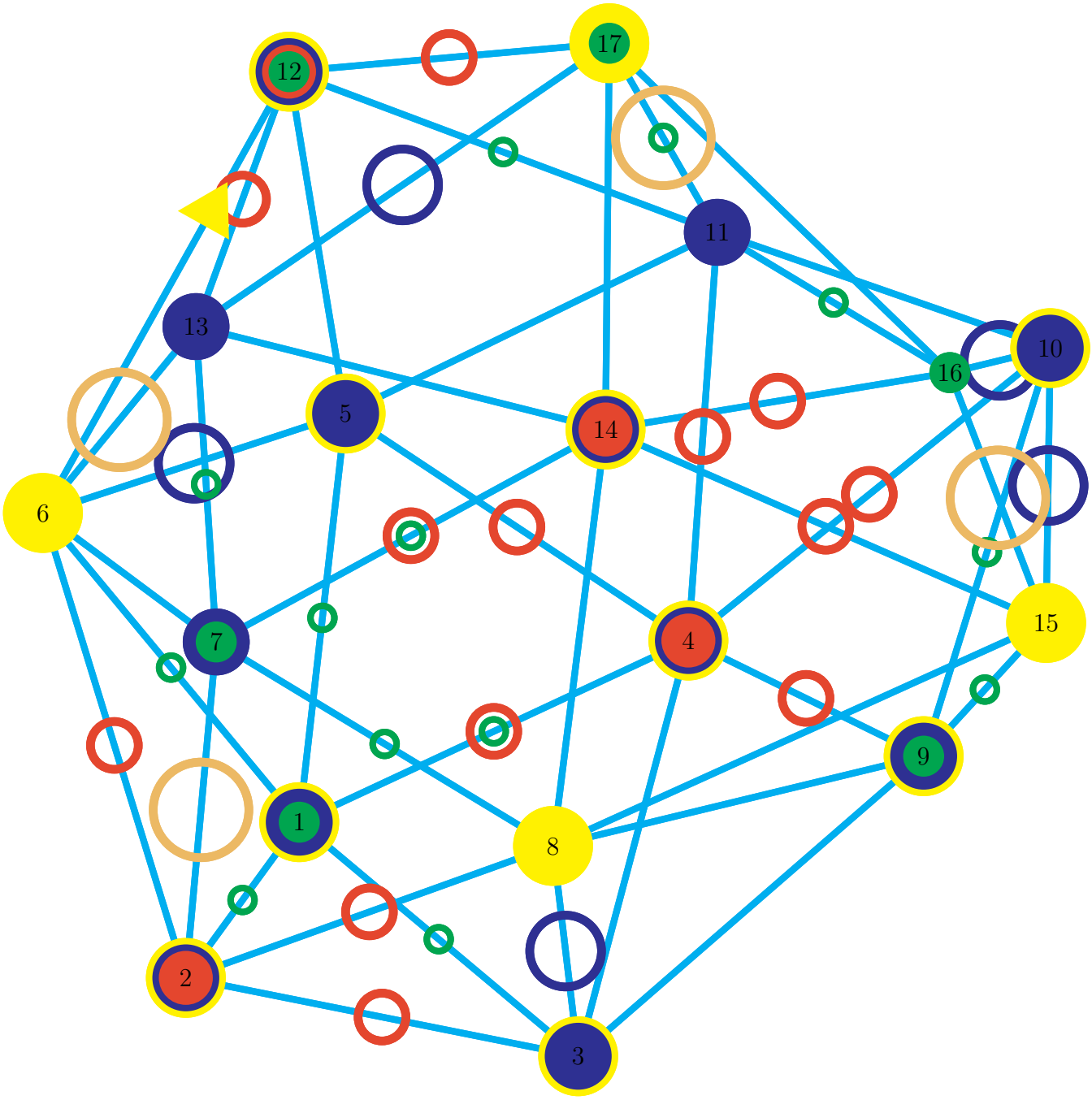

Figure 72: Instructions: 223: place edge 6→12 Yellow ArrowR

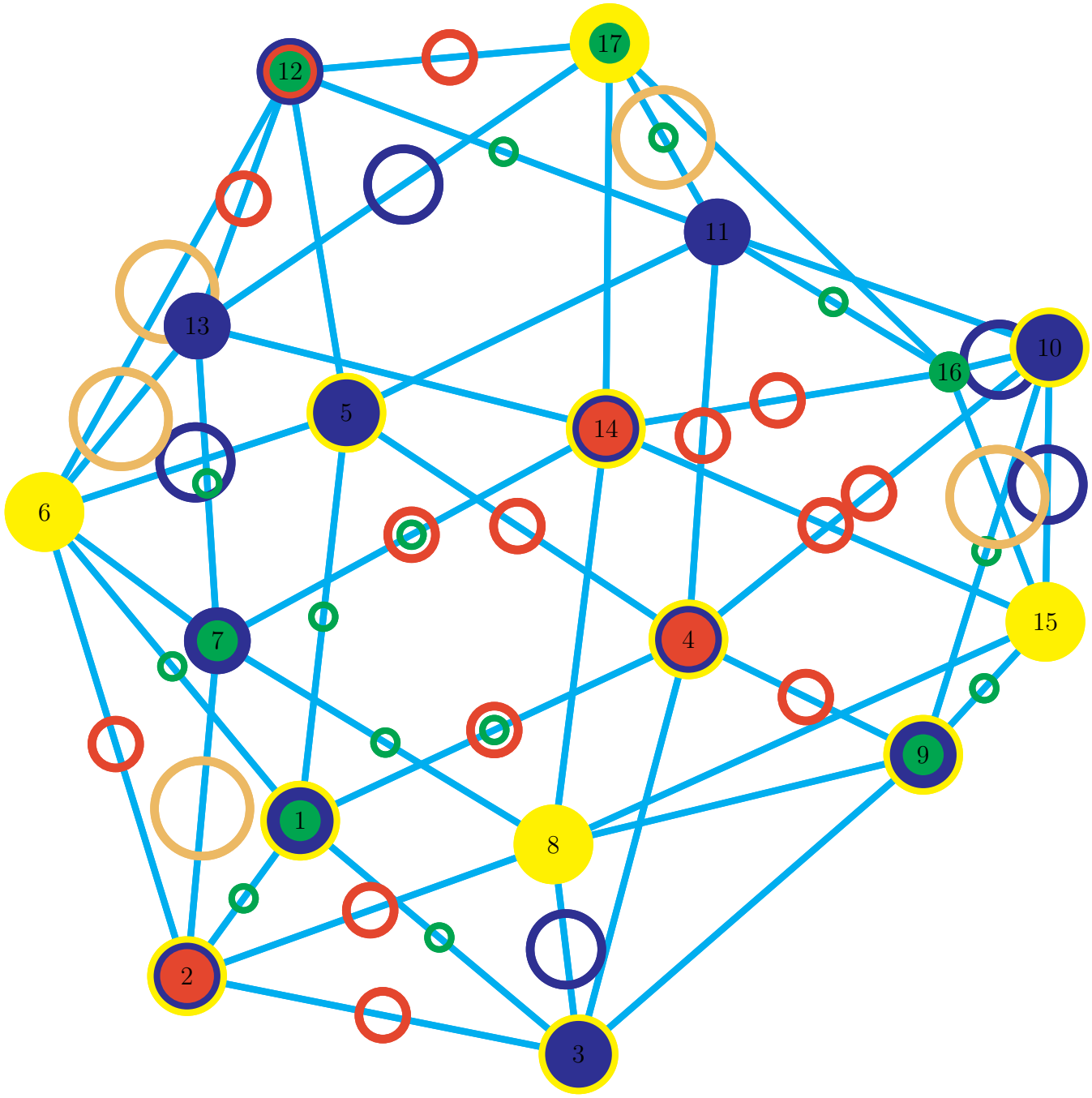

Figure 73: Instructions: 226: color edge 6→12 Yellow 227: uncolor vertex 12 Yellow

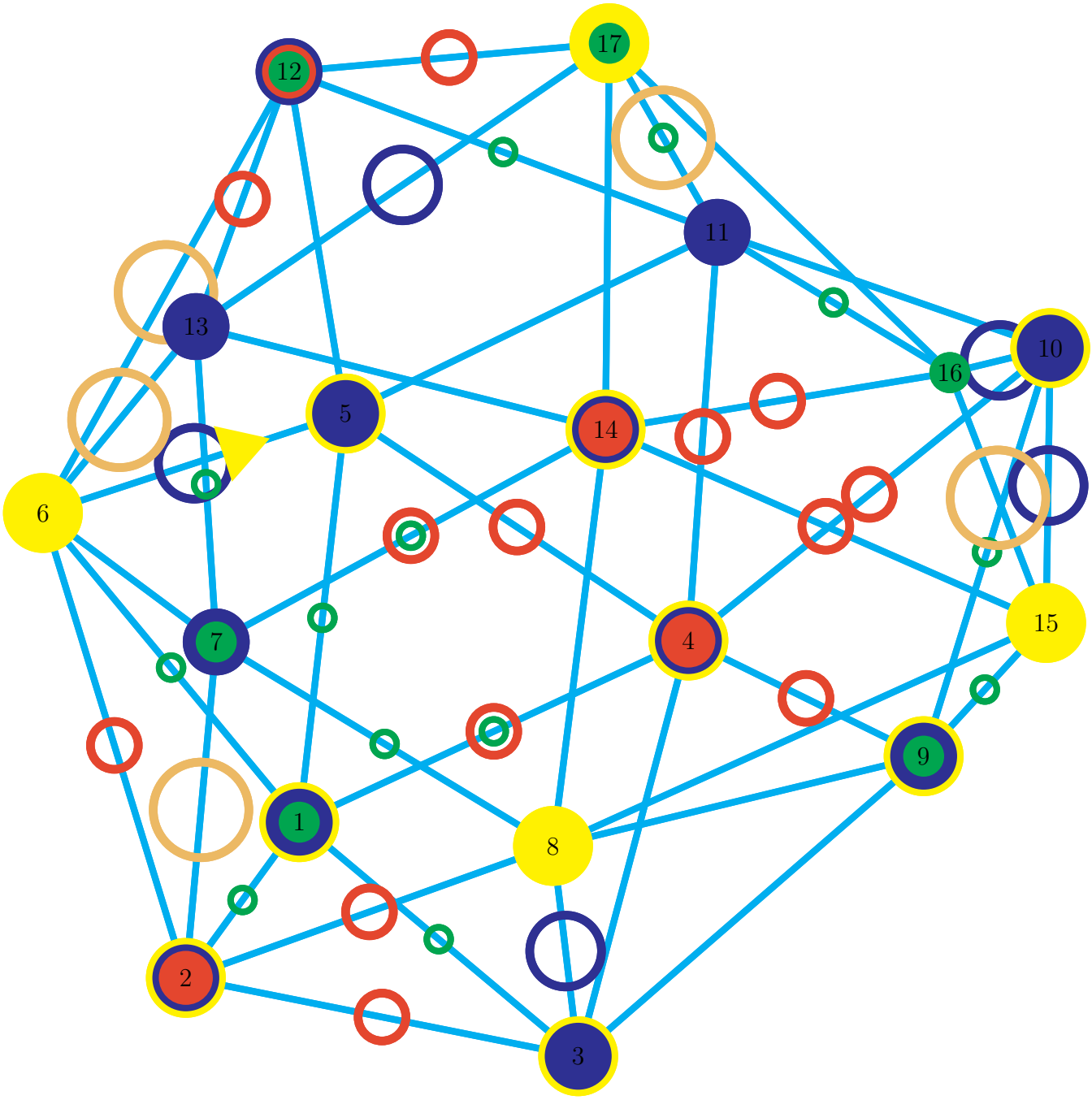

Figure 74: Instructions: 229: place edge 6→5 Yellow ArrowR

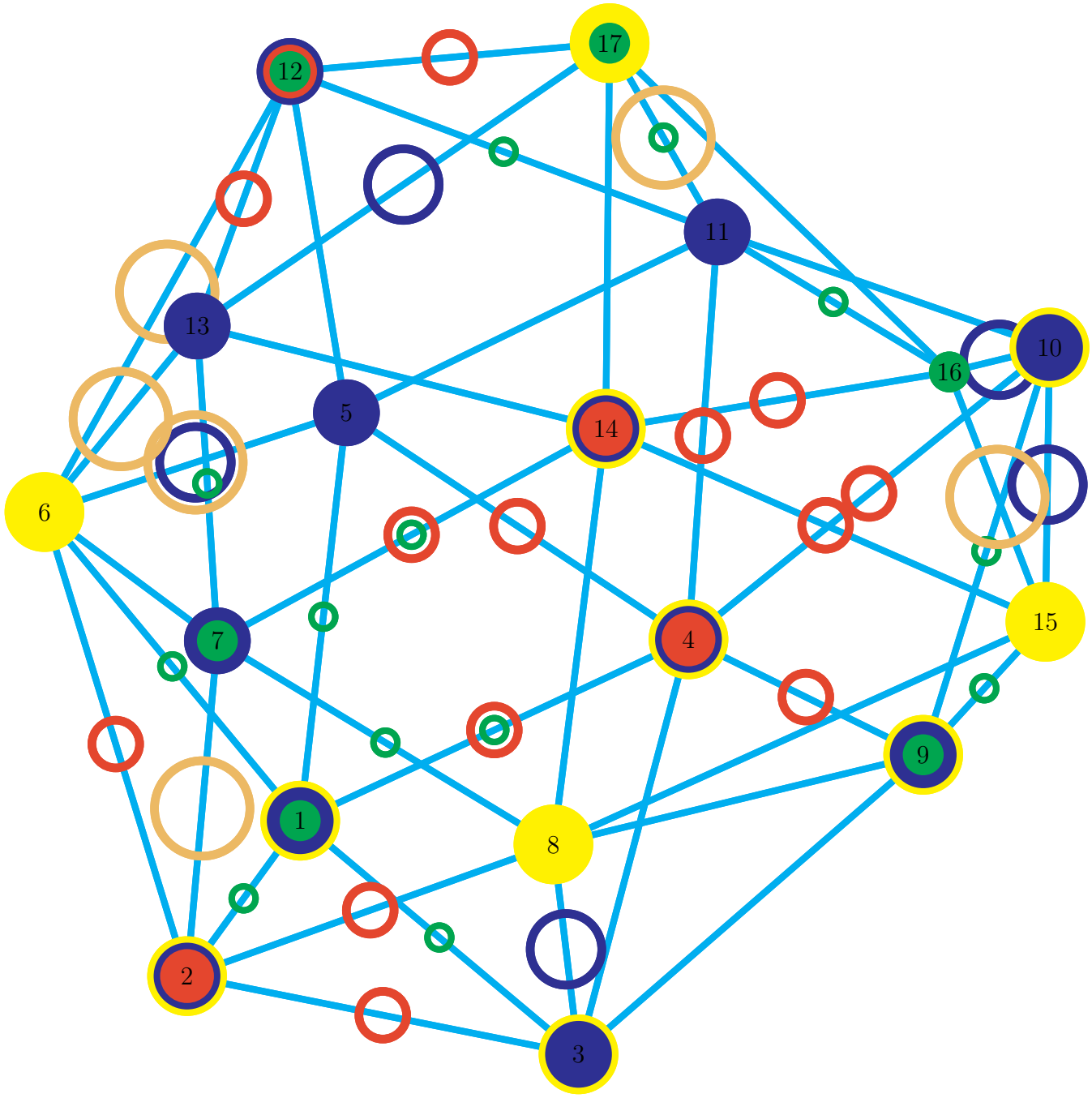

Figure 75: Instructions: 232: color edge 6→5 Yellow 233: uncolor vertex 5 Yellow

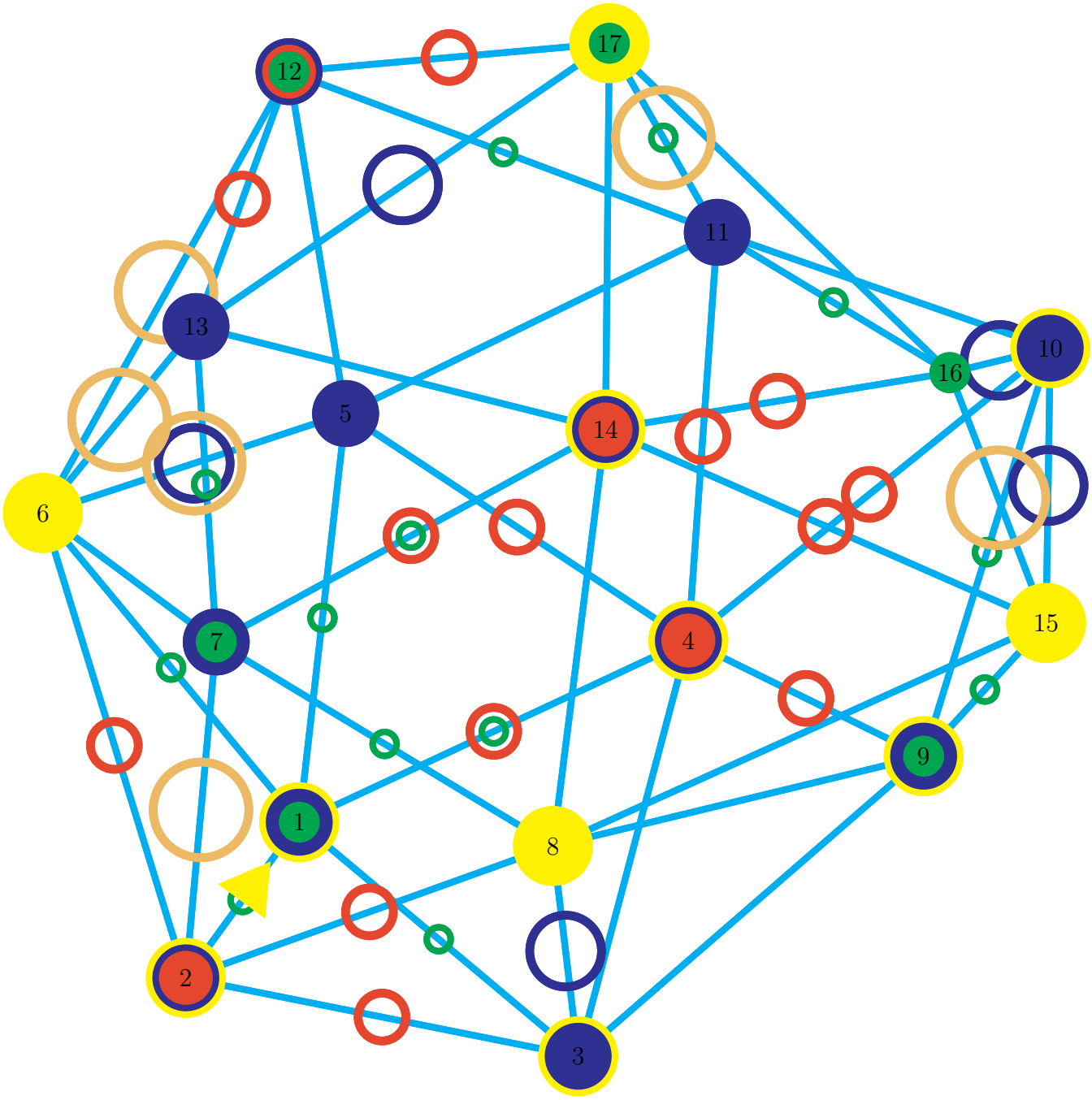

Figure 76: Instructions: 235: place edge 2→1 Yellow ArrowR

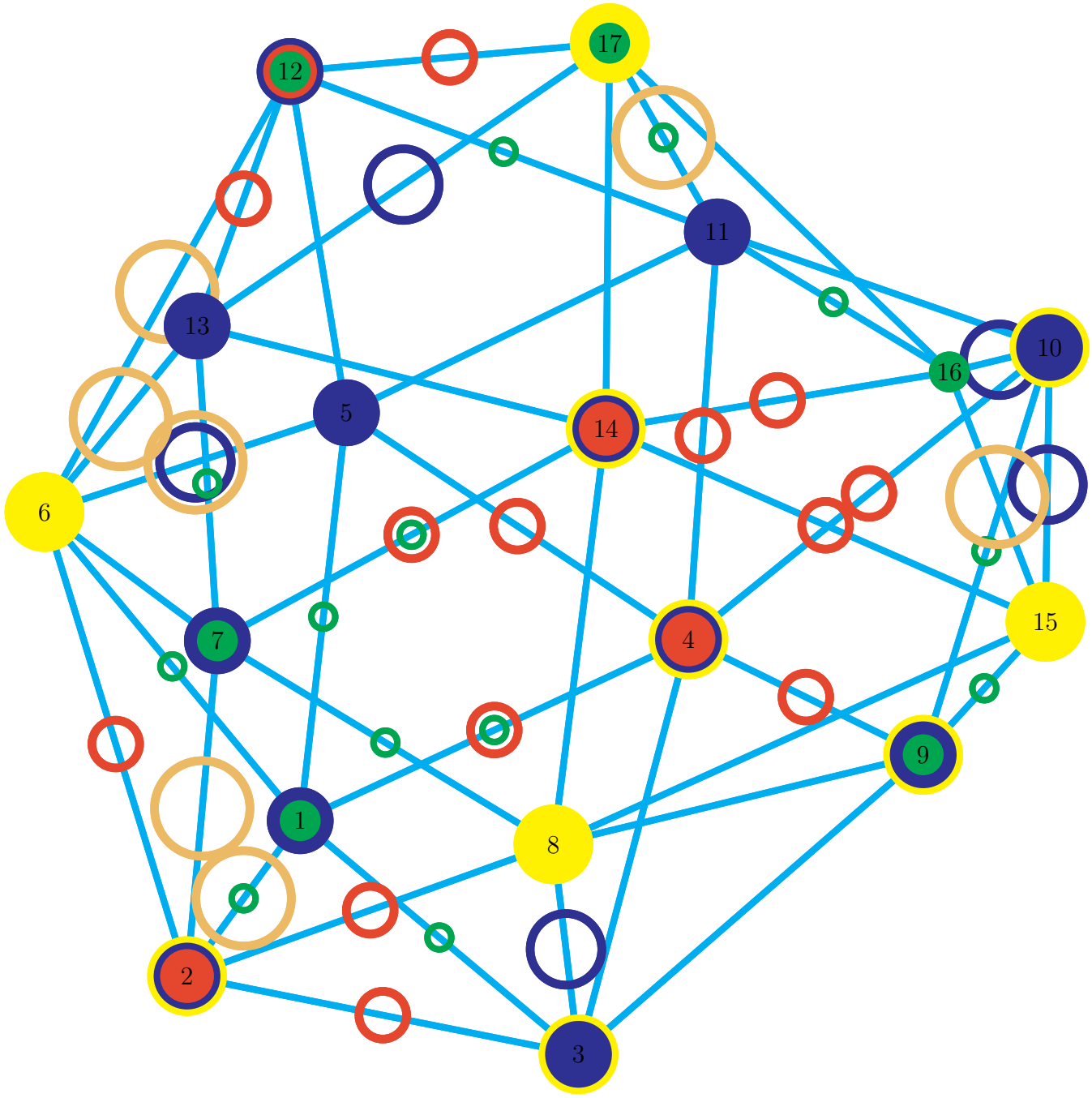

Figure 77: Instructions: 238: color edge 2→1 Yellow 239: uncolor vertex 1 Yellow

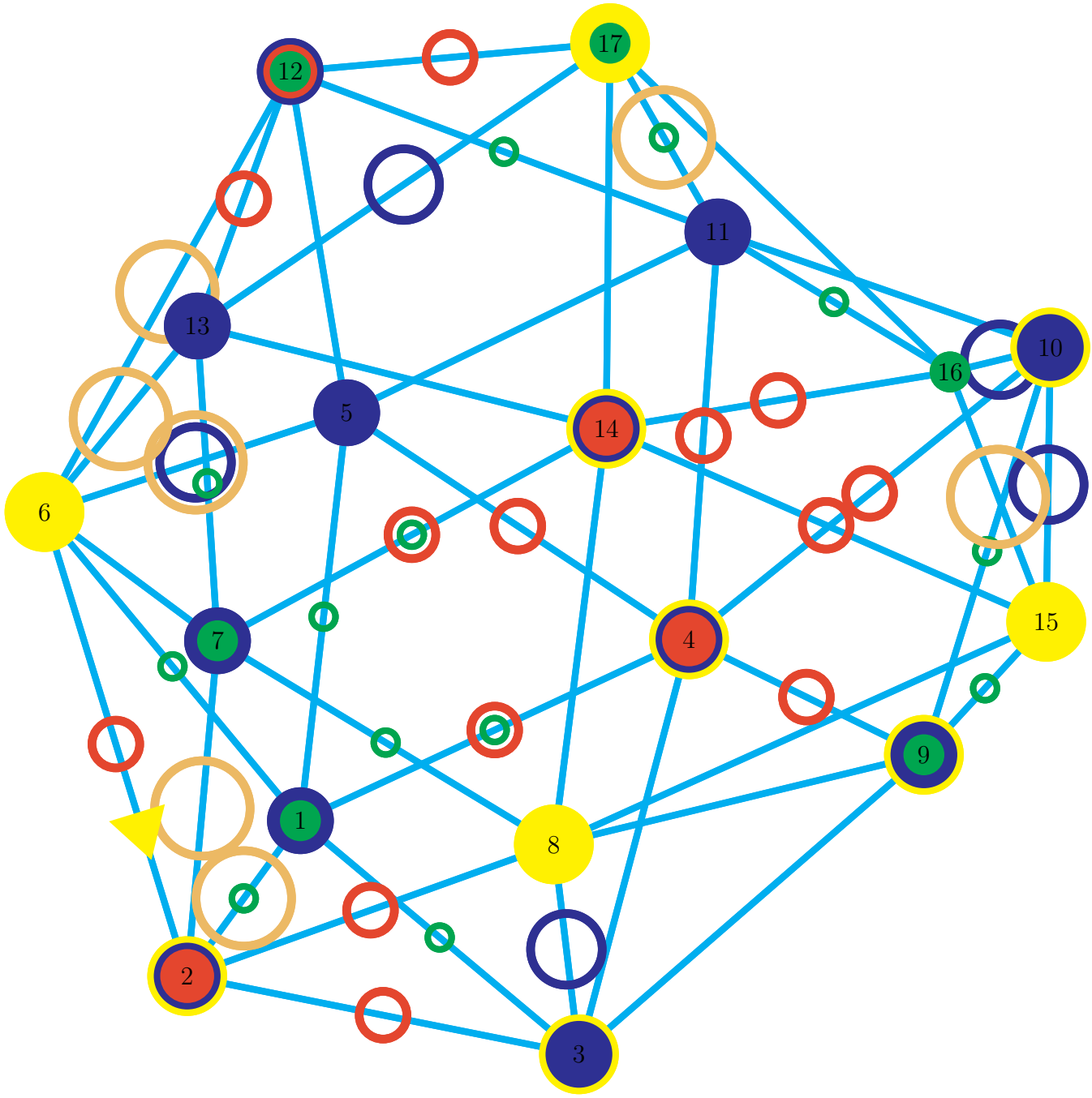

Figure 78: Instructions: 241: place edge 6→2 Yellow ArrowR

A yellow arrow has been placed on edge 6→2 indicating removal of the yellow checker from vertex 2. This will create a new problem.

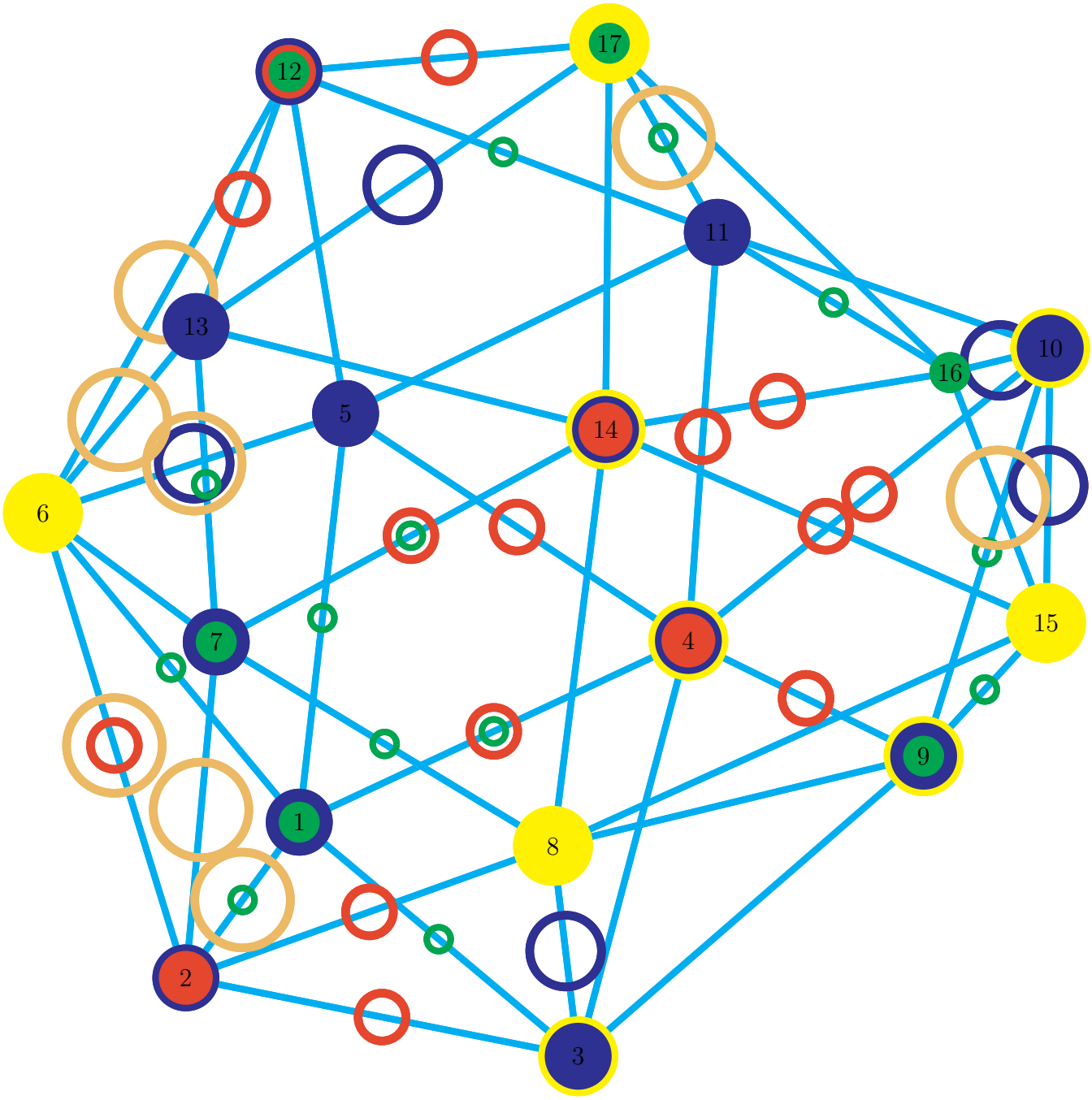

Figure 79: Instructions: 244: color edge 6→2 Yellow 245: uncolor vertex 2 Yellow

The yellow checker has been removed from vertex 2 and a yellow circle has been placed on edge 2-6. Now property A obtains but property B does not for there are yellow circles on edges 1-2 and 2-7 but no yellow checkers on vertices 1, 2 or 7. However the situation is easily remedied as follows.

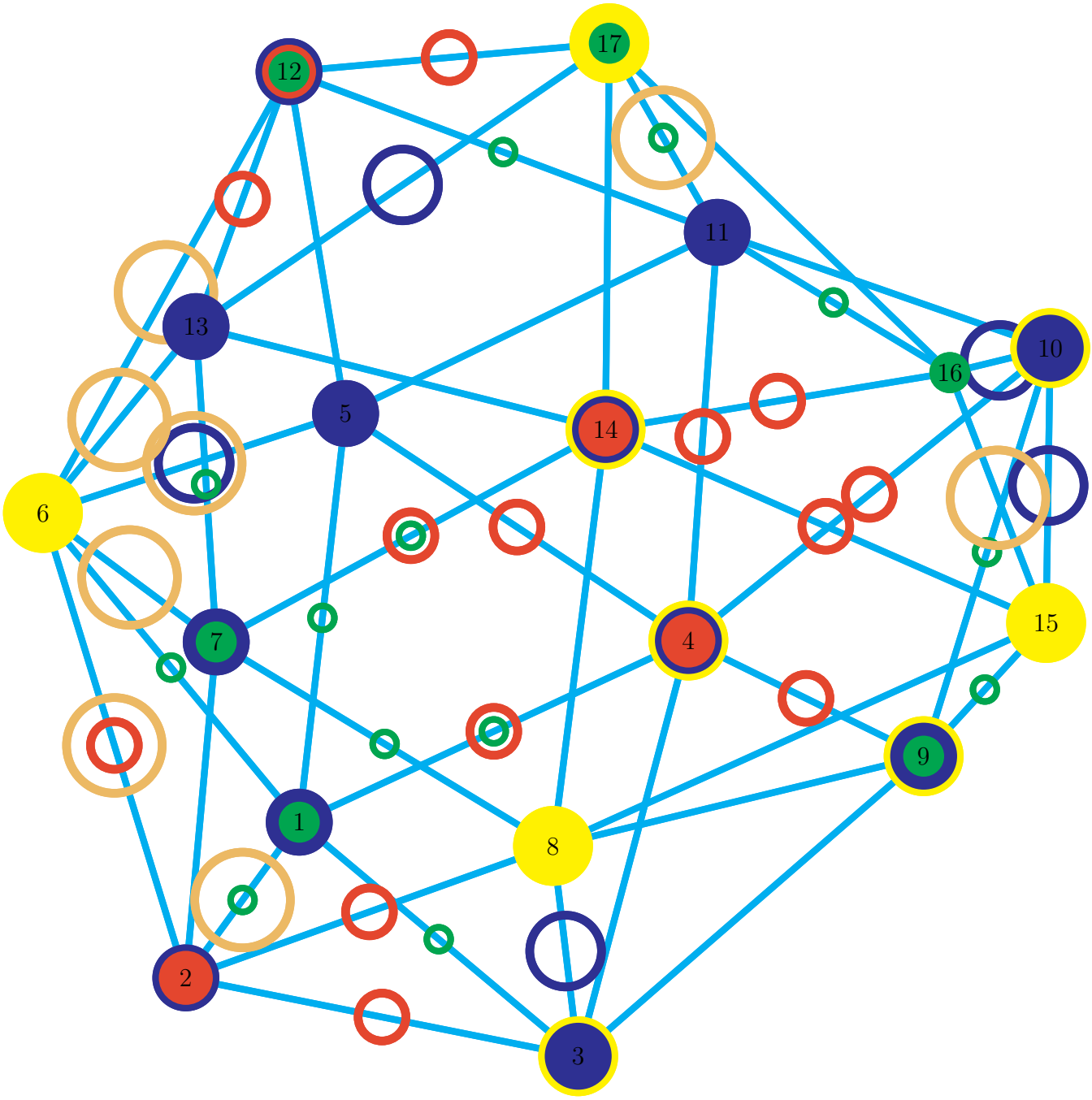

Figure 80: Instructions: 249: color edge 6→7 Yellow

The yellow circle on edge 2-7 has been flipped to edge 6-7.

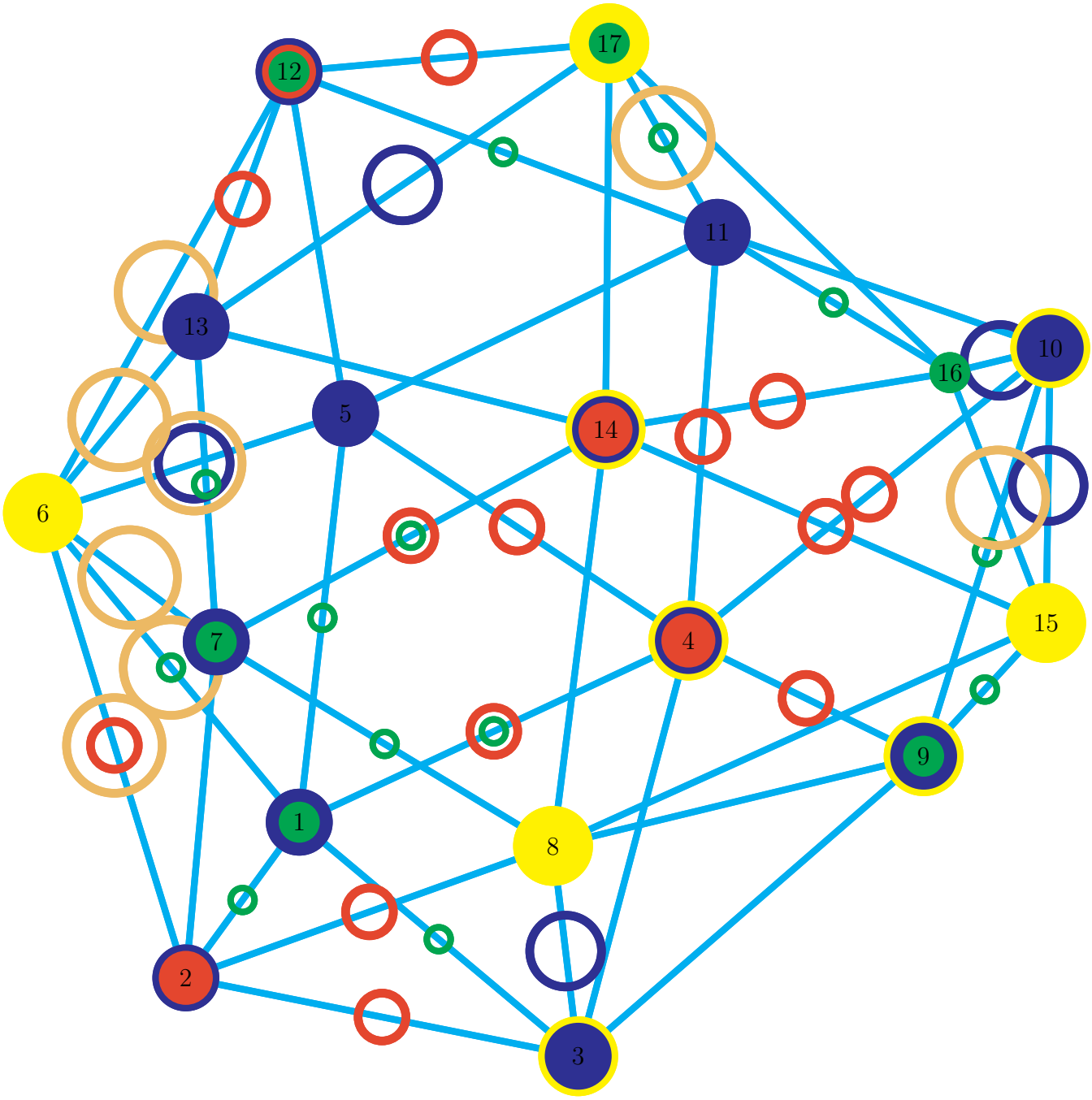

Figure 81: Instructions: 253: color edge 6→1 Yellow

The yellow circle on edge 1-2 has been flipped to edge 2-6 and property B is restored.

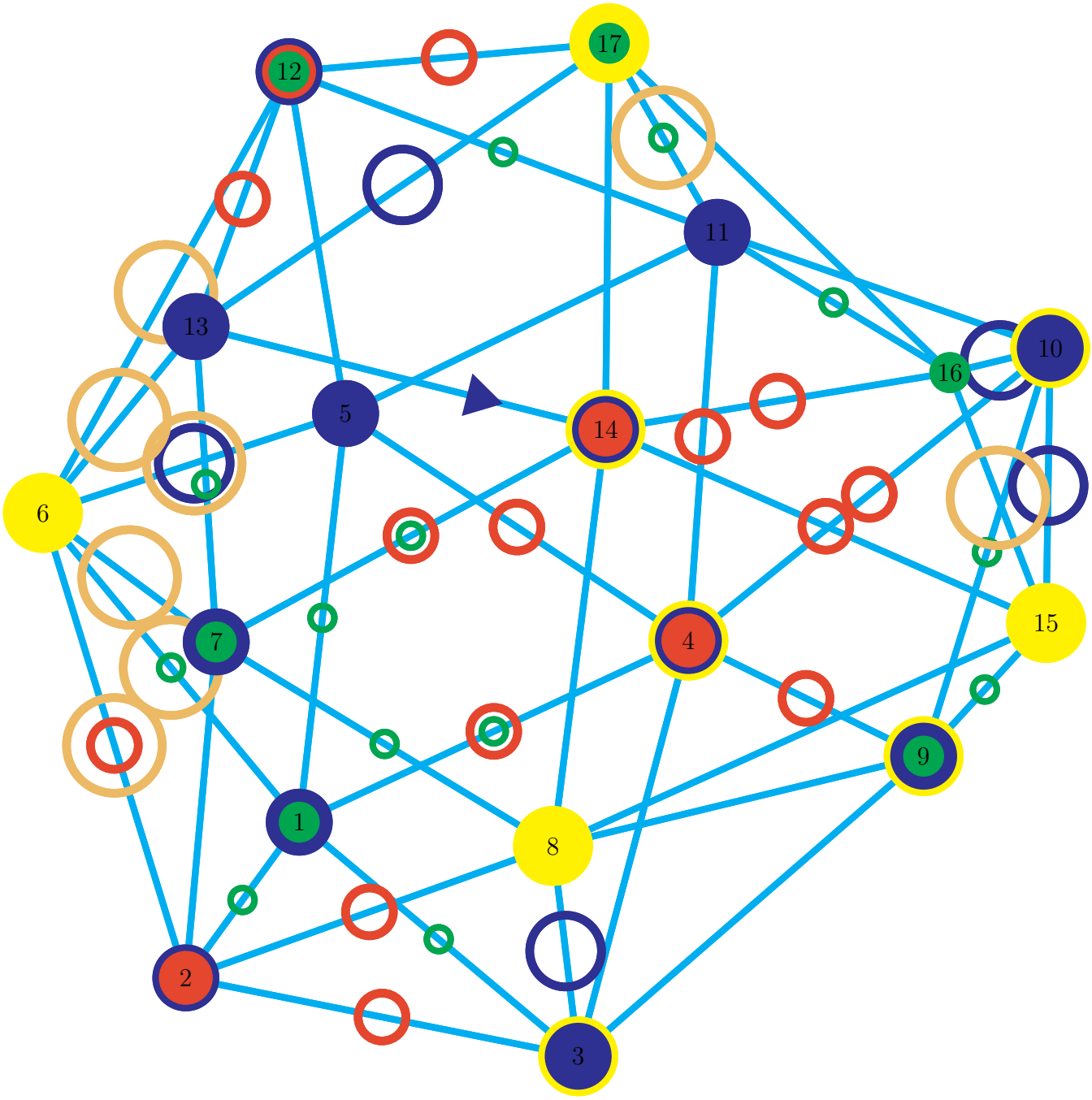

Figure 82: Instructions: 255: place edge 13→14 Blue ArrowR

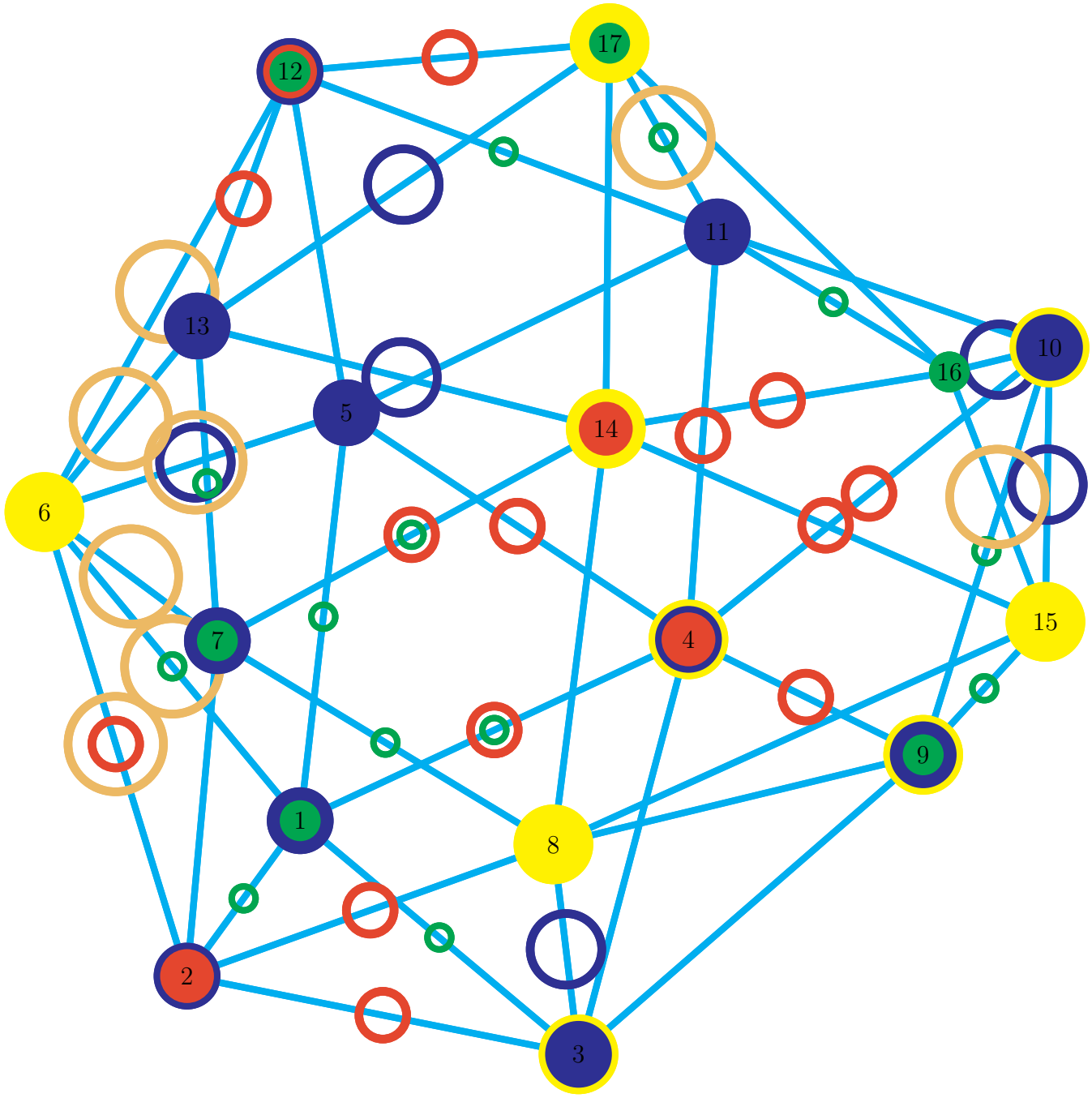

Figure 83: Instructions: 258: color edge 13→14 Blue 259: uncolor vertex 14 Blue

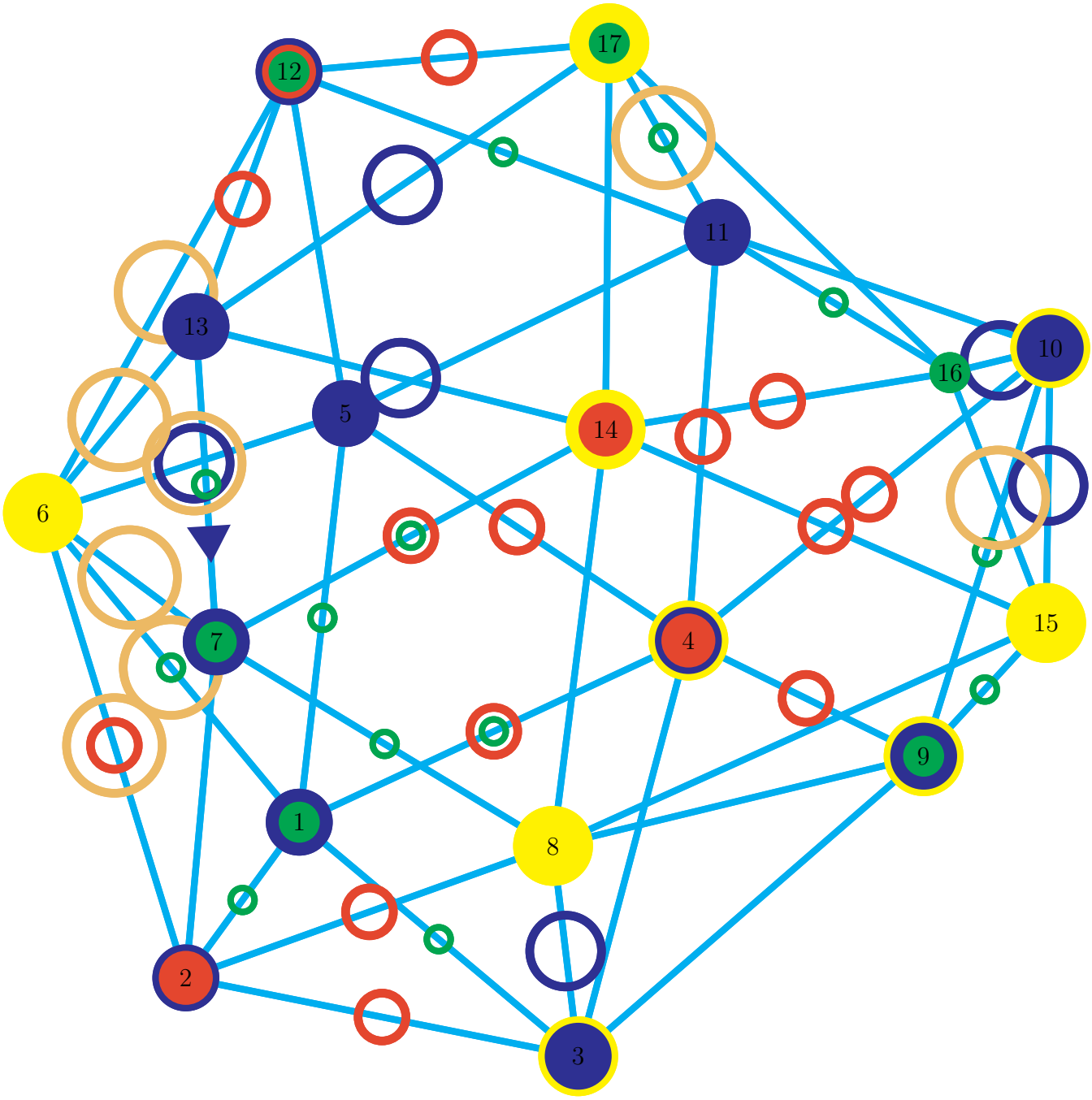

Figure 84: Instructions: 261: place edge 13→7 Blue ArrowR

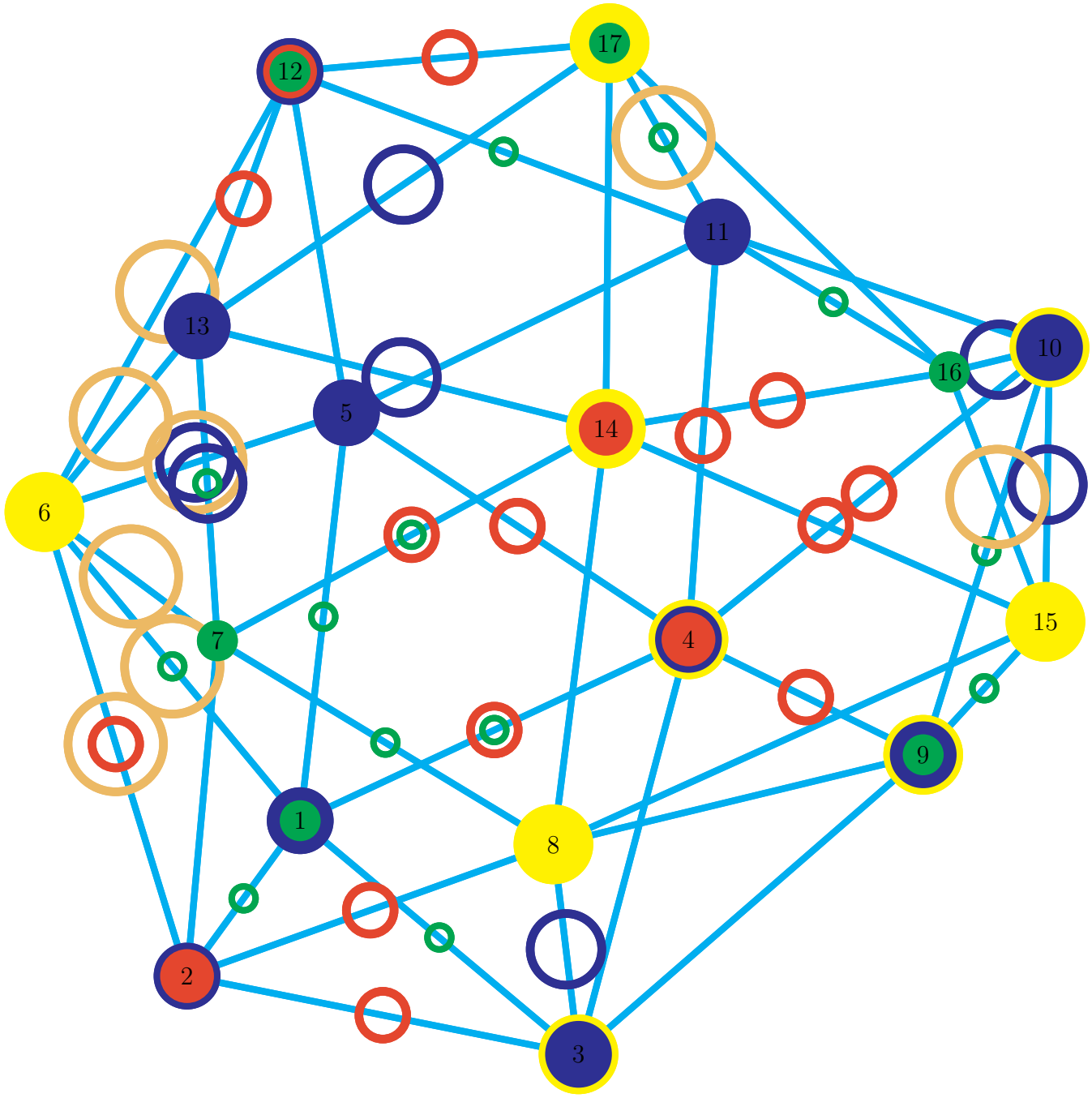

Figure 85: Instructions: 264: color edge 13→7 Blue 265: uncolor vertex 7 Blue

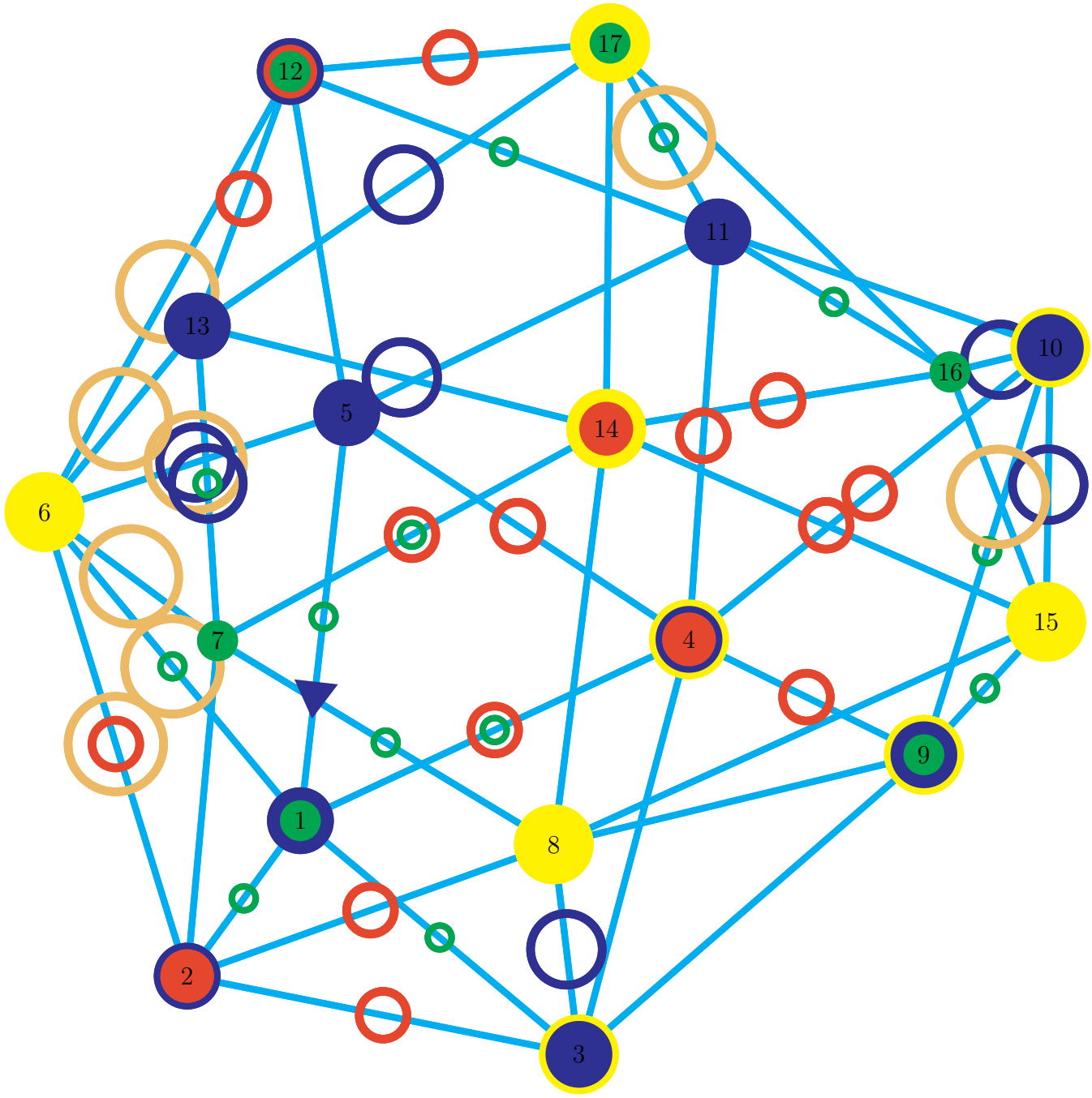

Figure 86: Instructions: 267: place edge 5→1 Blue ArrowR

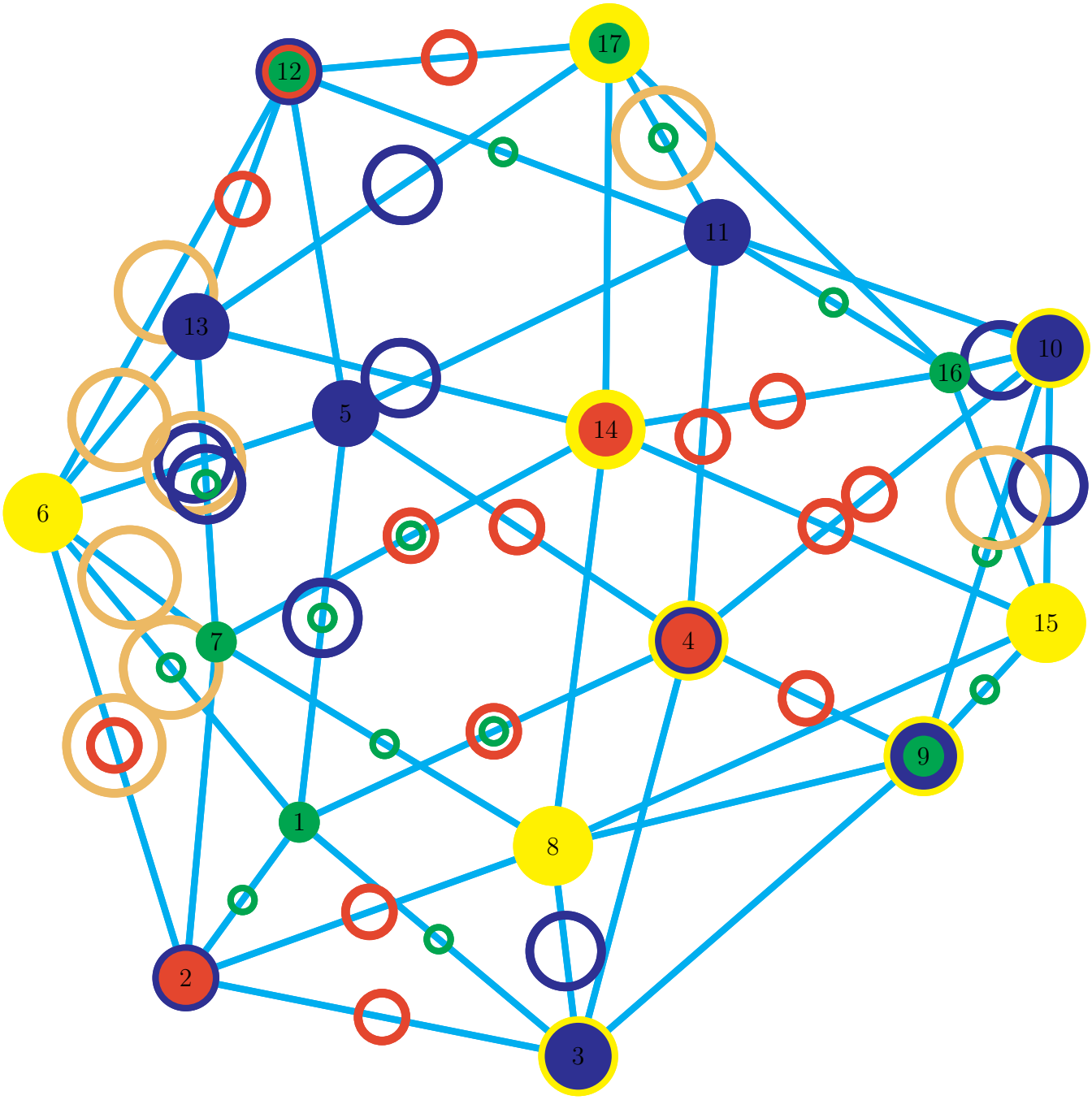

Figure 87: Instructions: 270: color edge 5→1 Blue 271: uncolor vertex 1 Blue

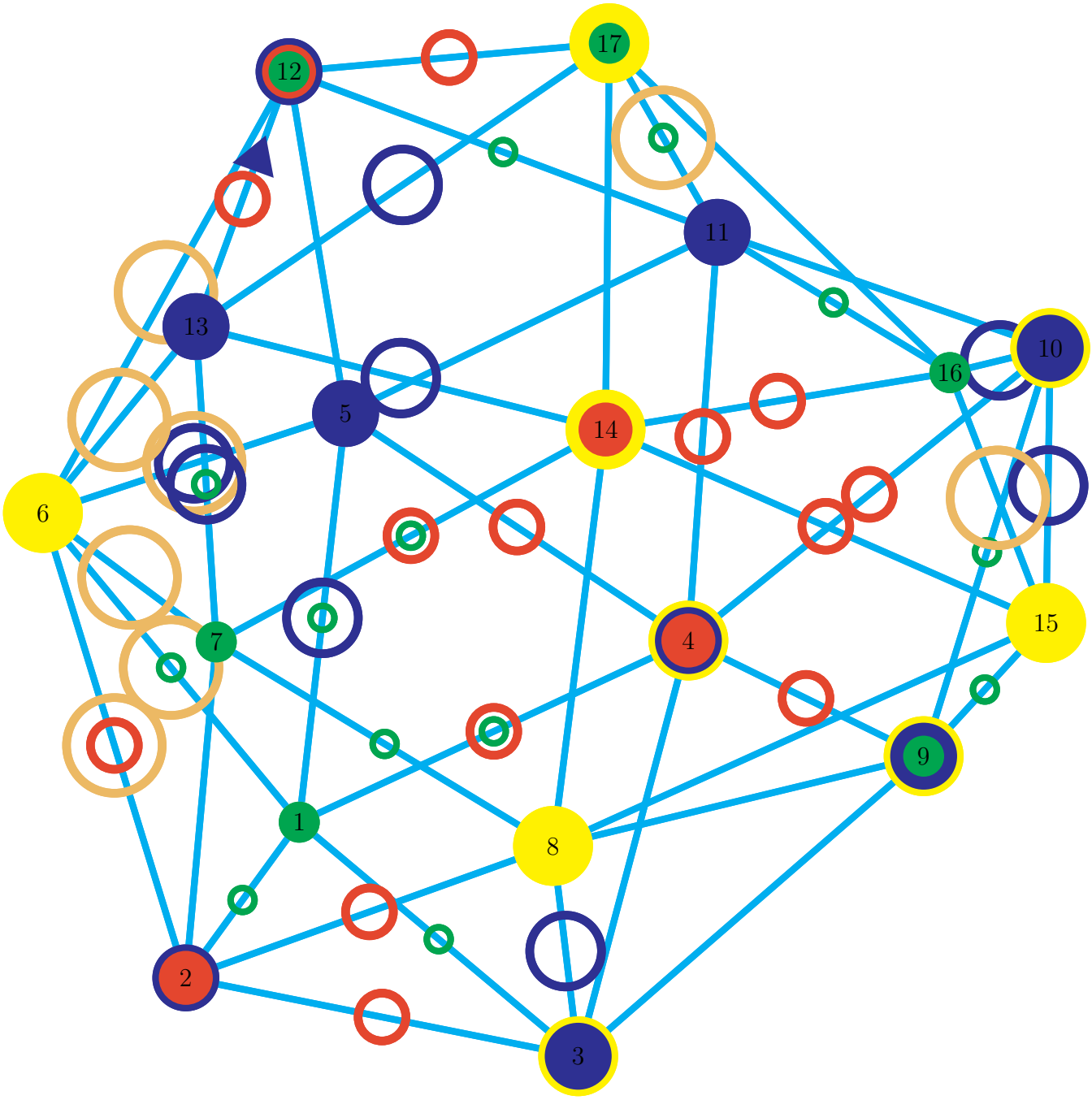

Figure 88: Instructions: 273: place edge 13→12 Blue ArrowR

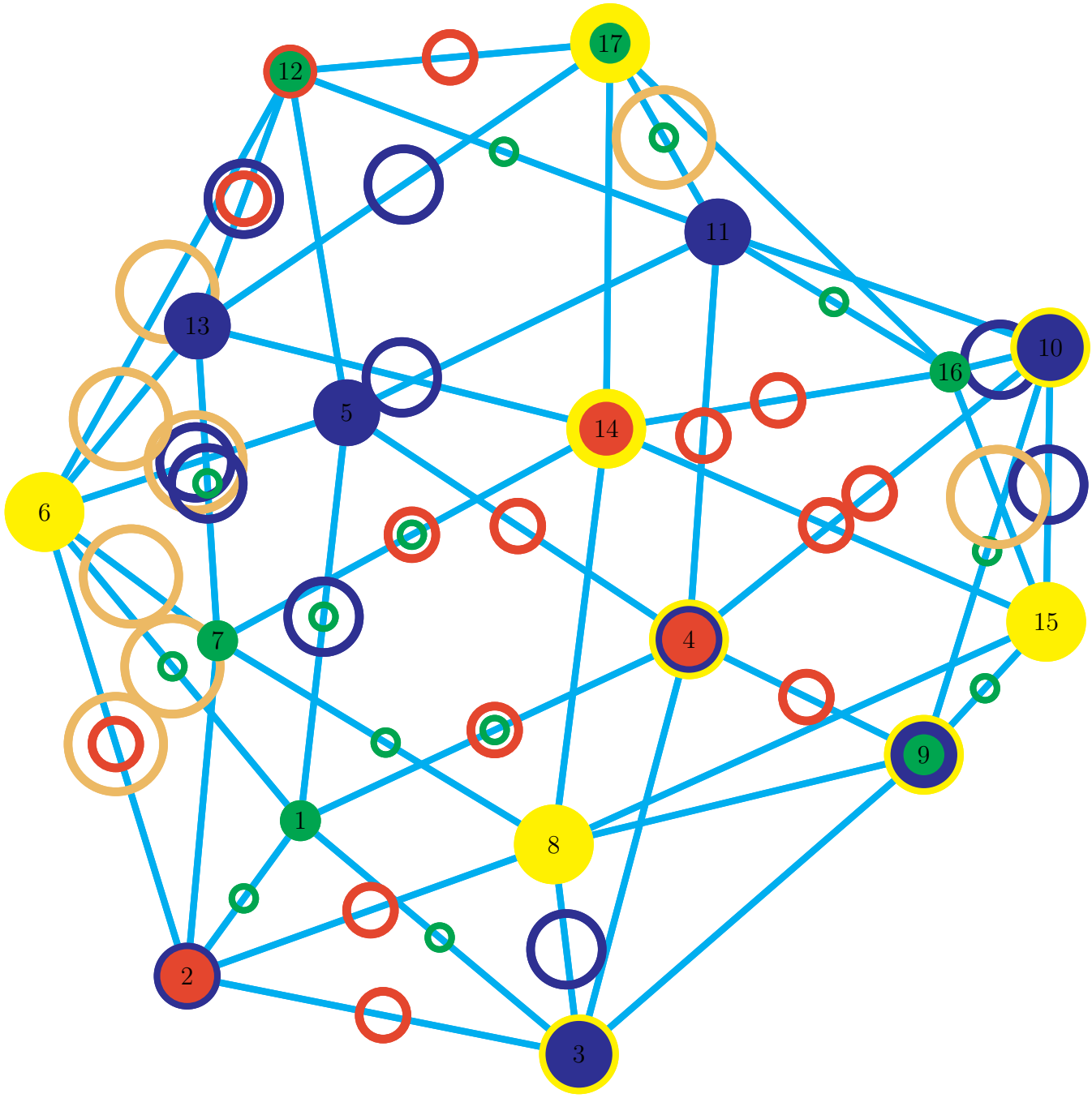

Figure 89: Instructions: 276: color edge 13→12 Blue 277: uncolor vertex 12 Blue

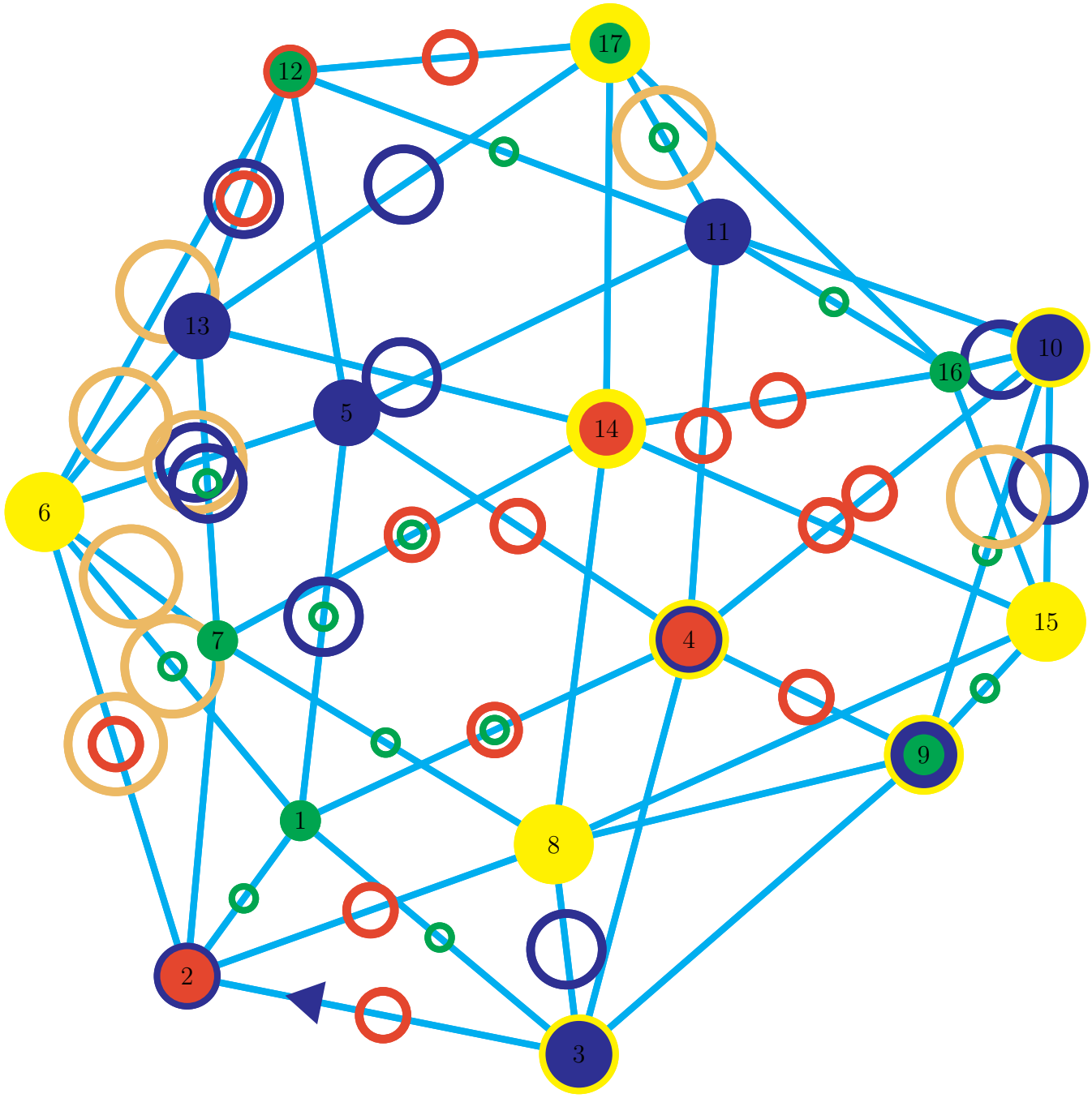

Figure 90: Instructions: 279: place edge 3→2 Blue ArrowR

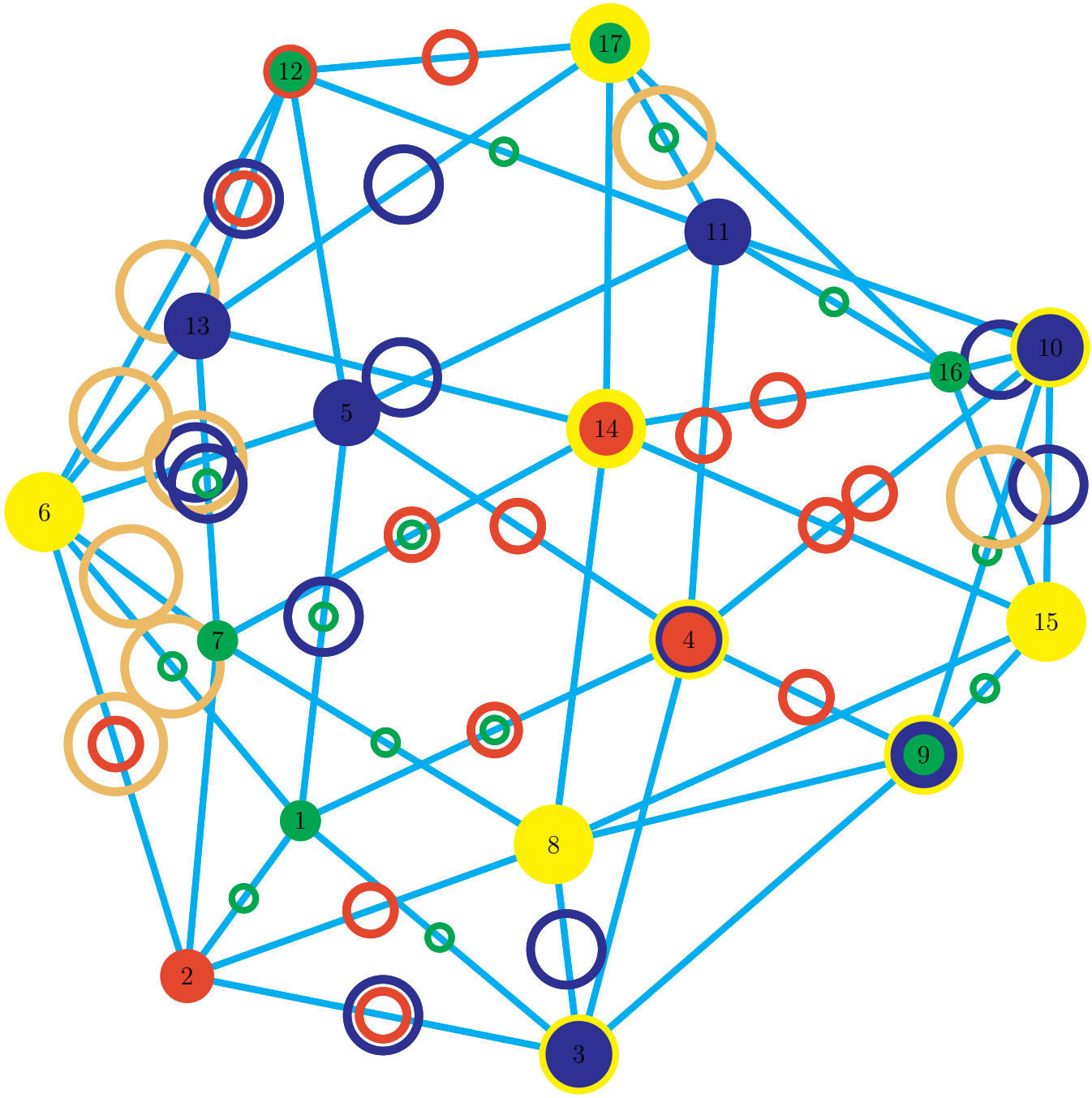

Figure 91: Instructions: 282: color edge 3→2 Blue 283: uncolor vertex 2 Blue

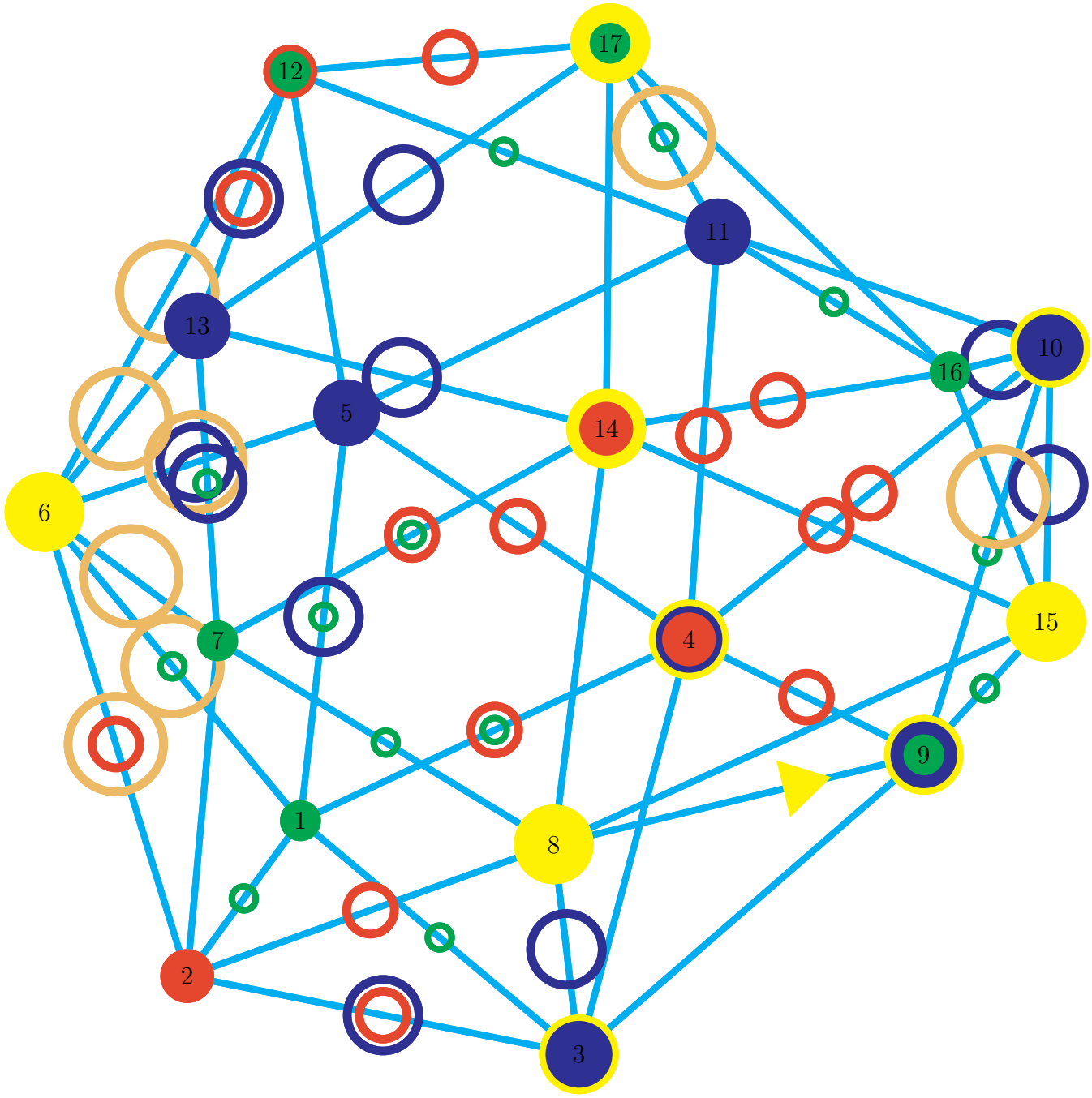

Figure 92: Instructions: 285: place edge 8→9 Yellow ArrowR

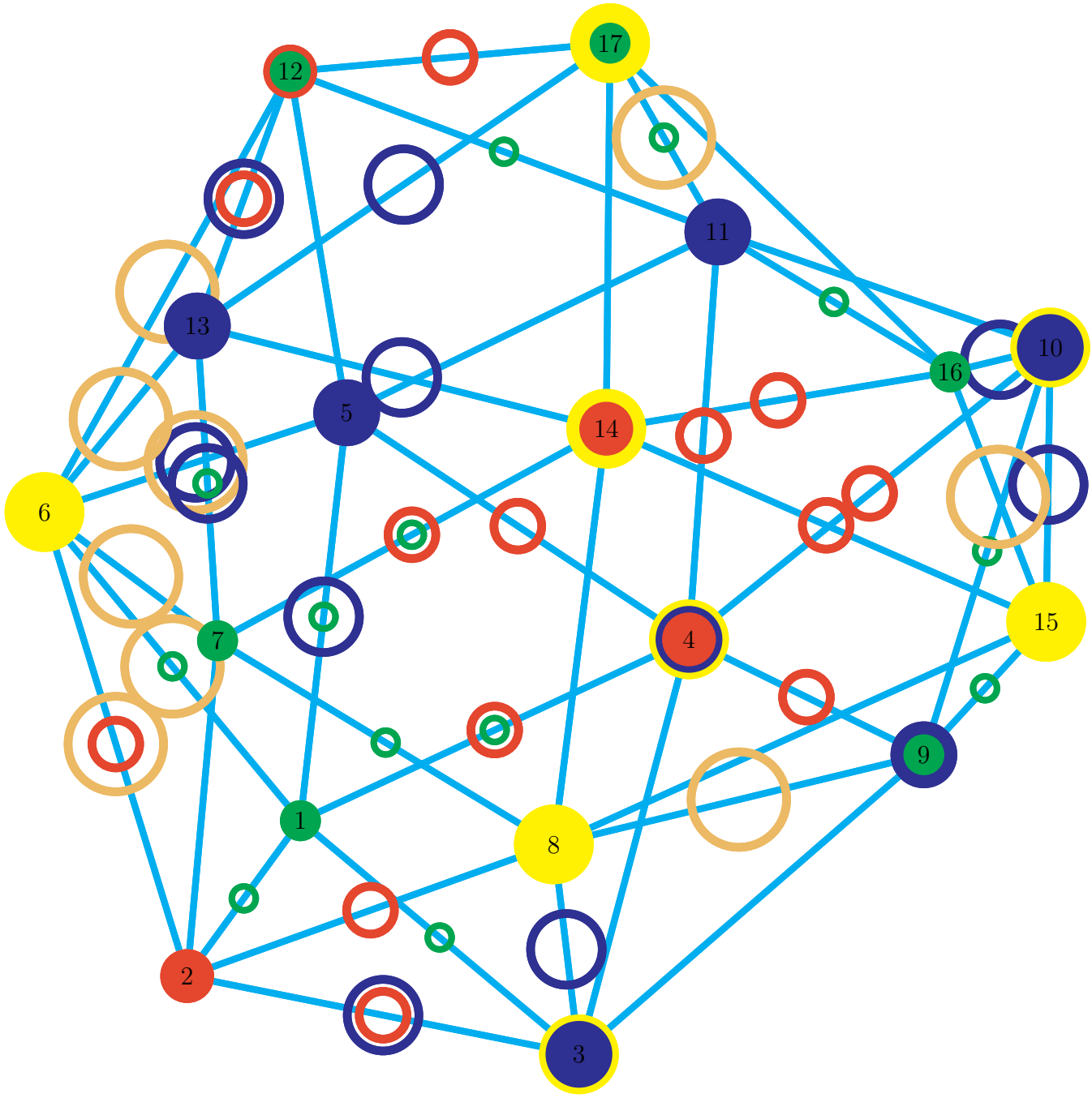

Figure 93: Instructions: 288: color edge 8→9 Yellow 289: uncolor vertex 9 Yellow

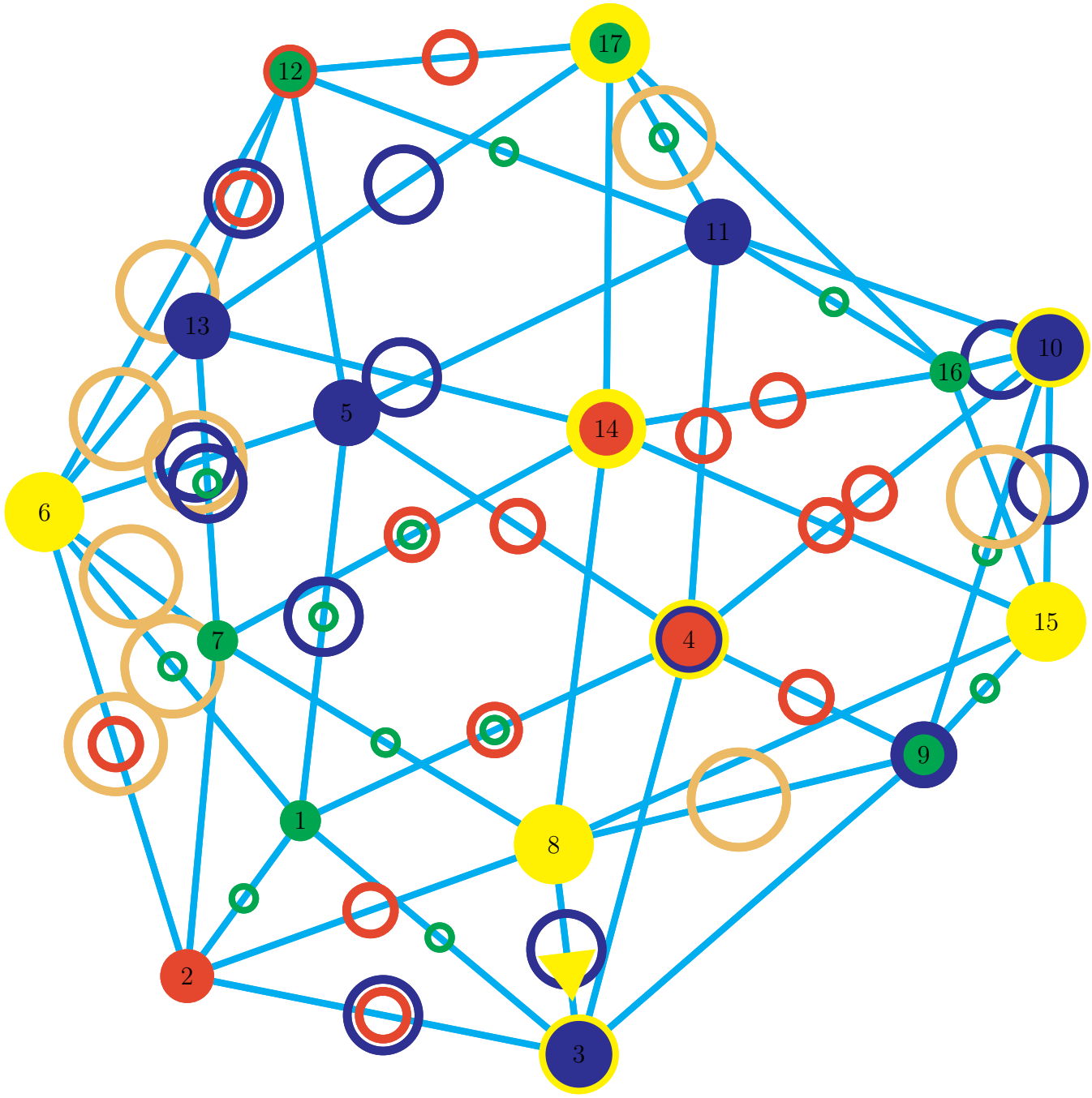

Figure 94: Instructions: 291: place edge 8→3 Yellow ArrowR

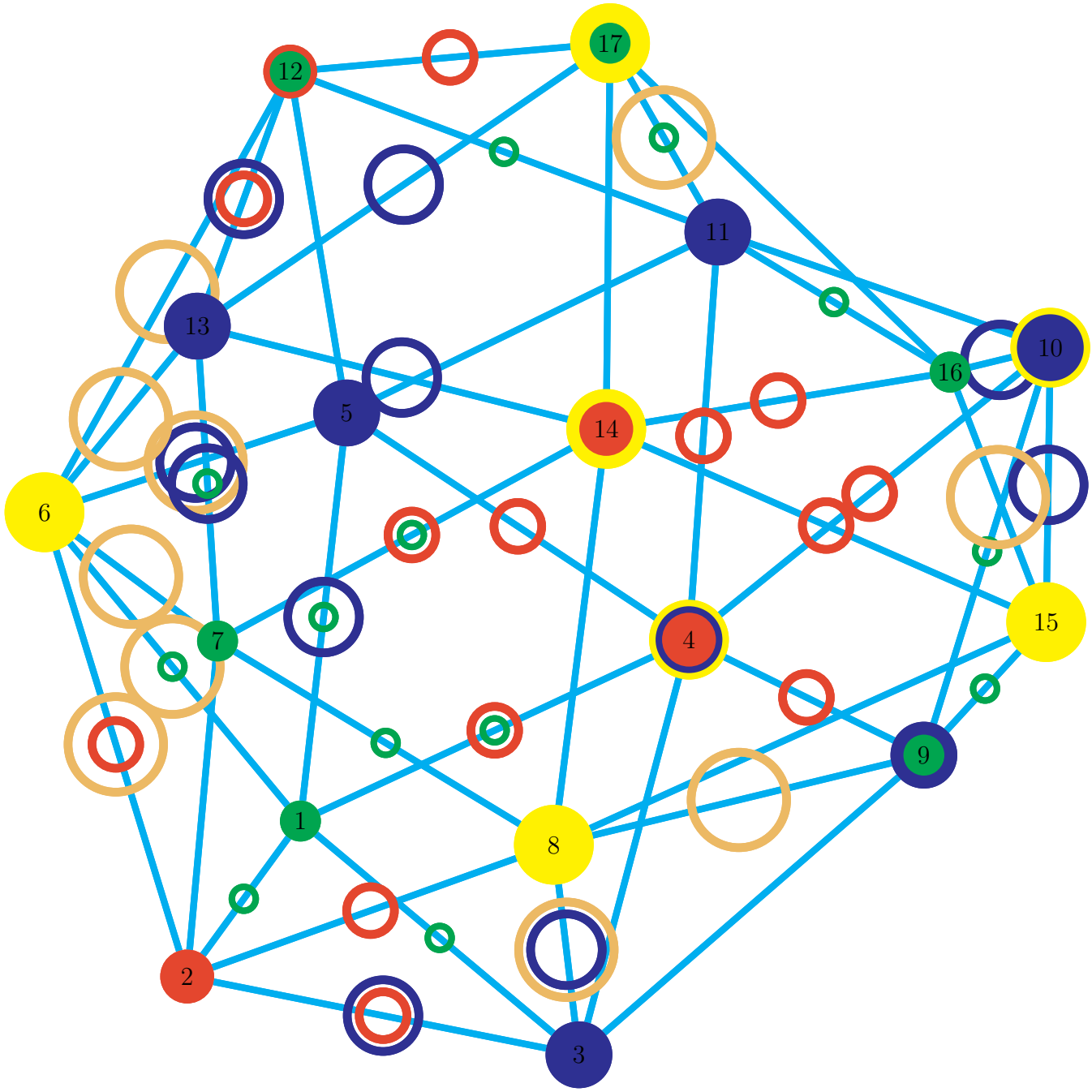

Figure 95: Instructions: 294: color edge 8→3 Yellow 295: uncolor vertex 3 Yellow

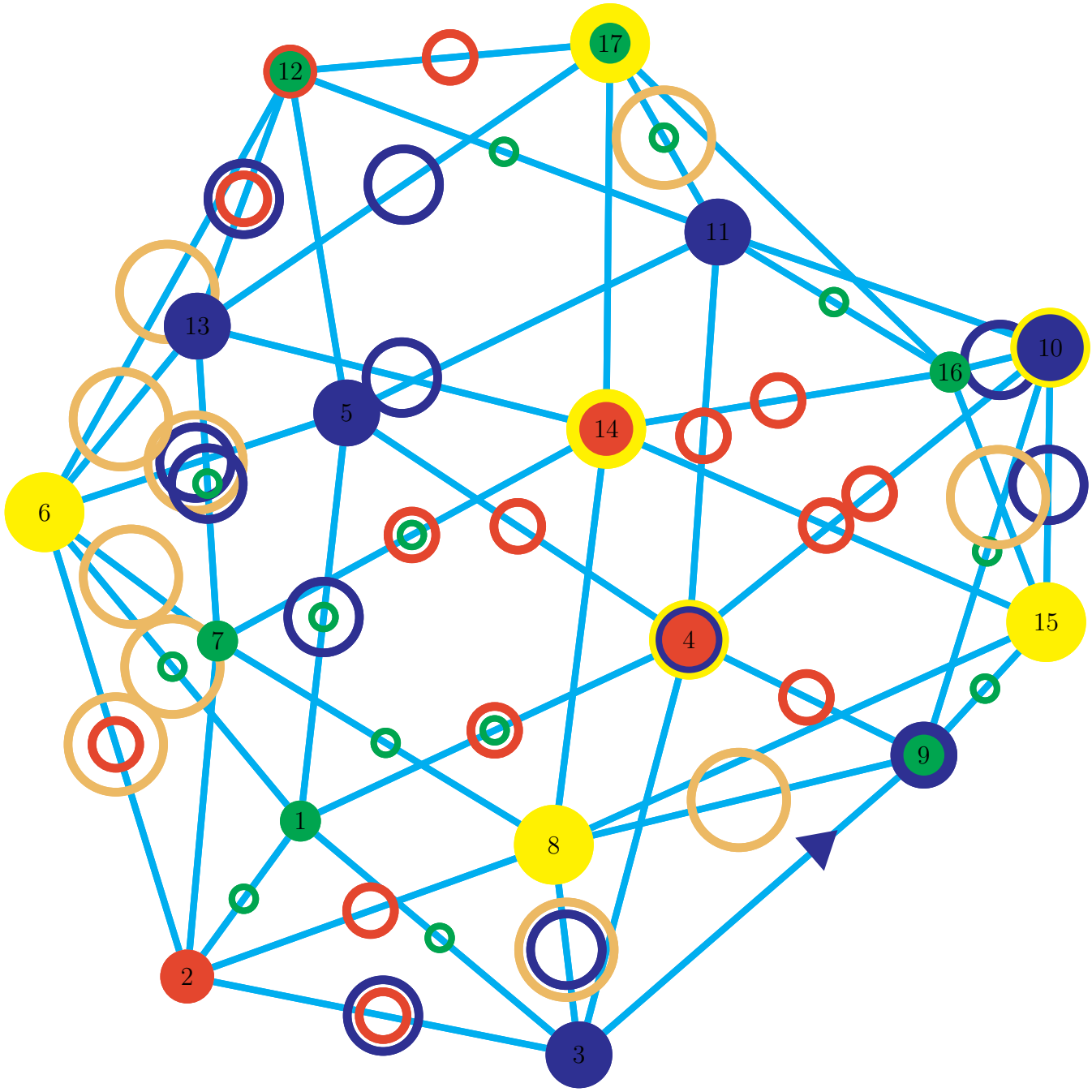

Figure 96: Instructions: 297: place edge 3→9 Blue ArrowR

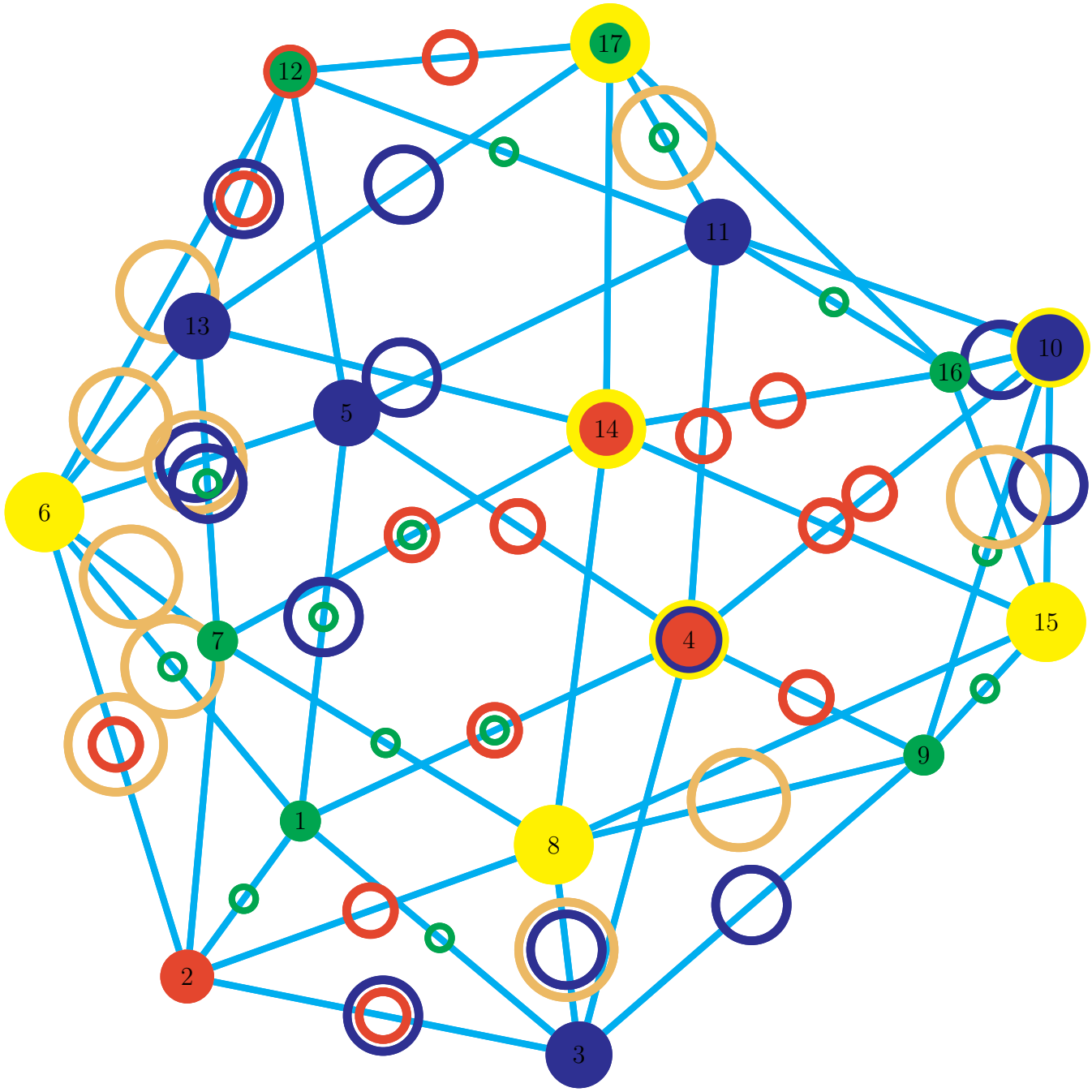

Figure 97: Instructions: 300: color edge 3→9 Blue 301: uncolor vertex 9 Blue

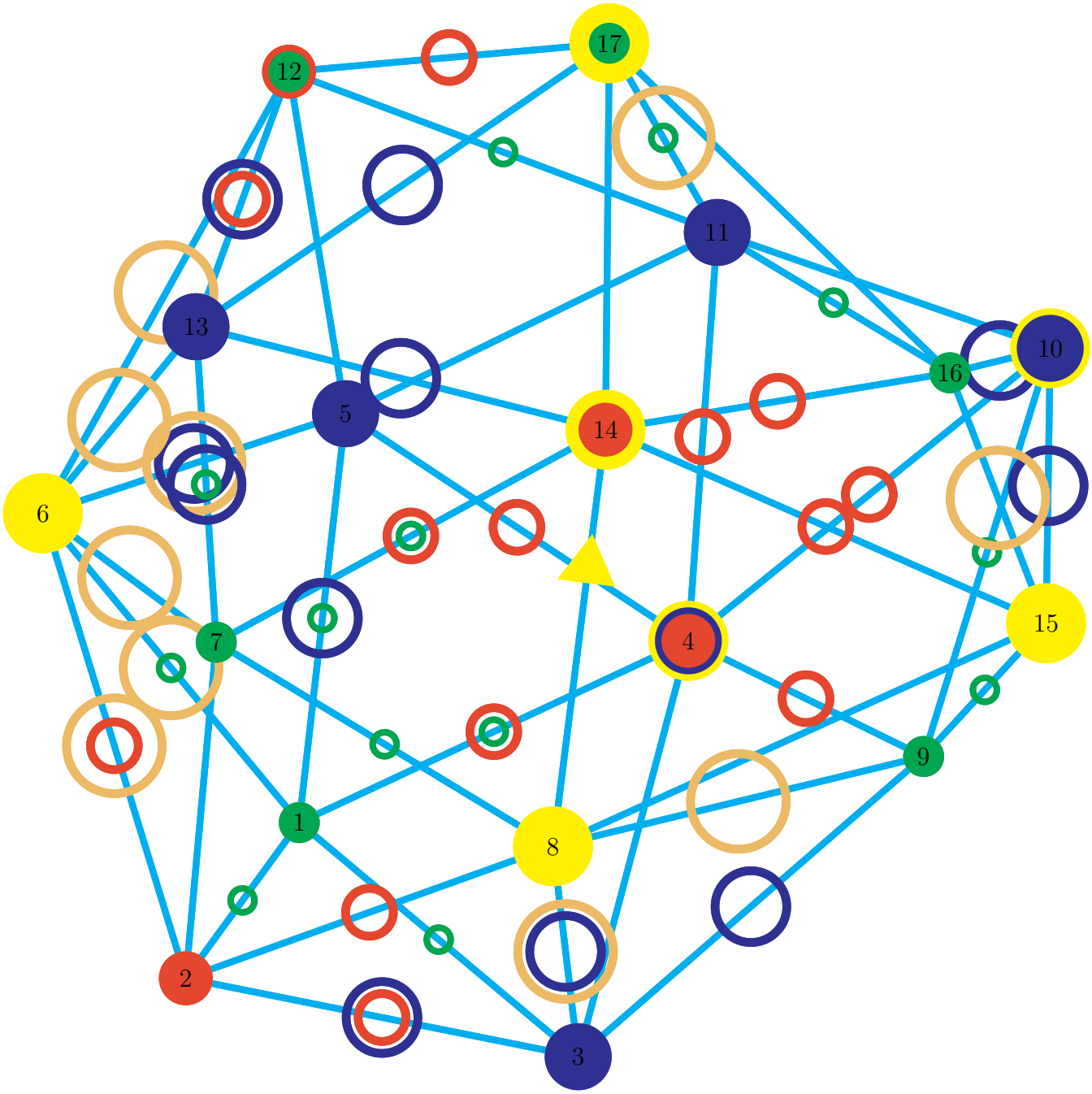

Figure 98: Instructions: 303: place edge 8→14 Yellow ArrowR

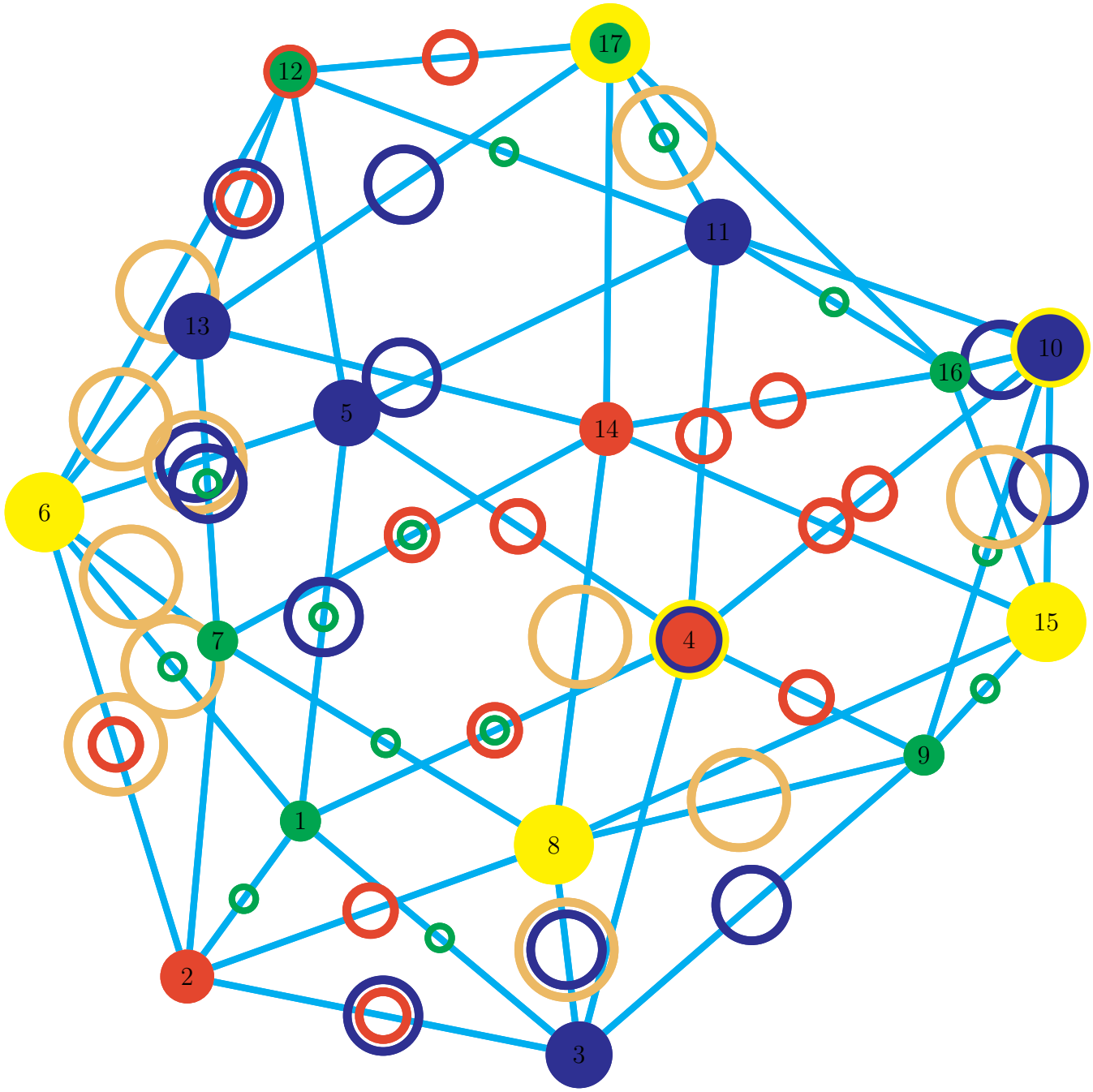

Figure 99: Instructions: 306: color edge 8→14 Yellow 307: uncolor vertex 14 Yellow

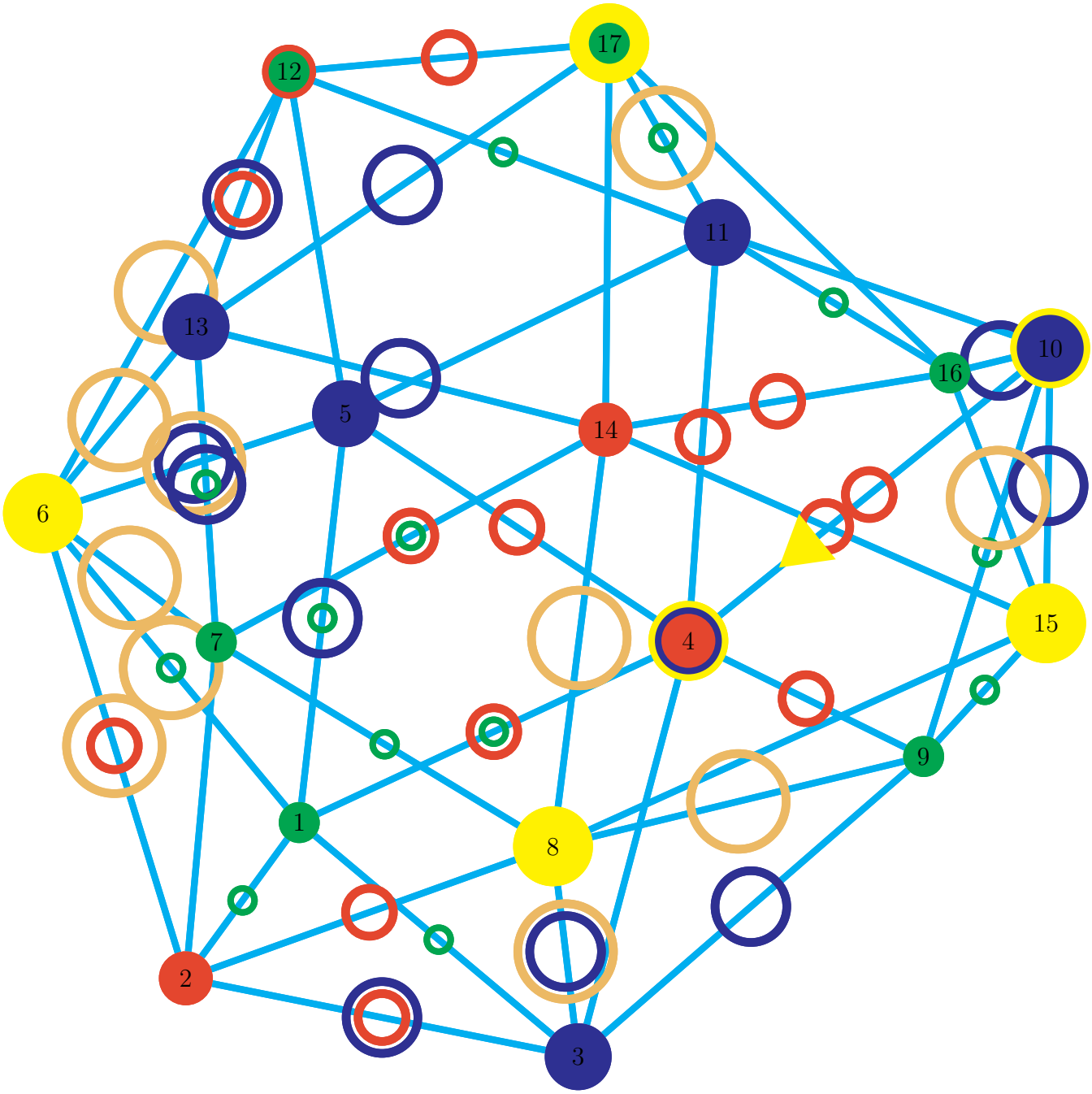

Figure 100: Instructions: 309: place edge 10→4 Yellow ArrowR

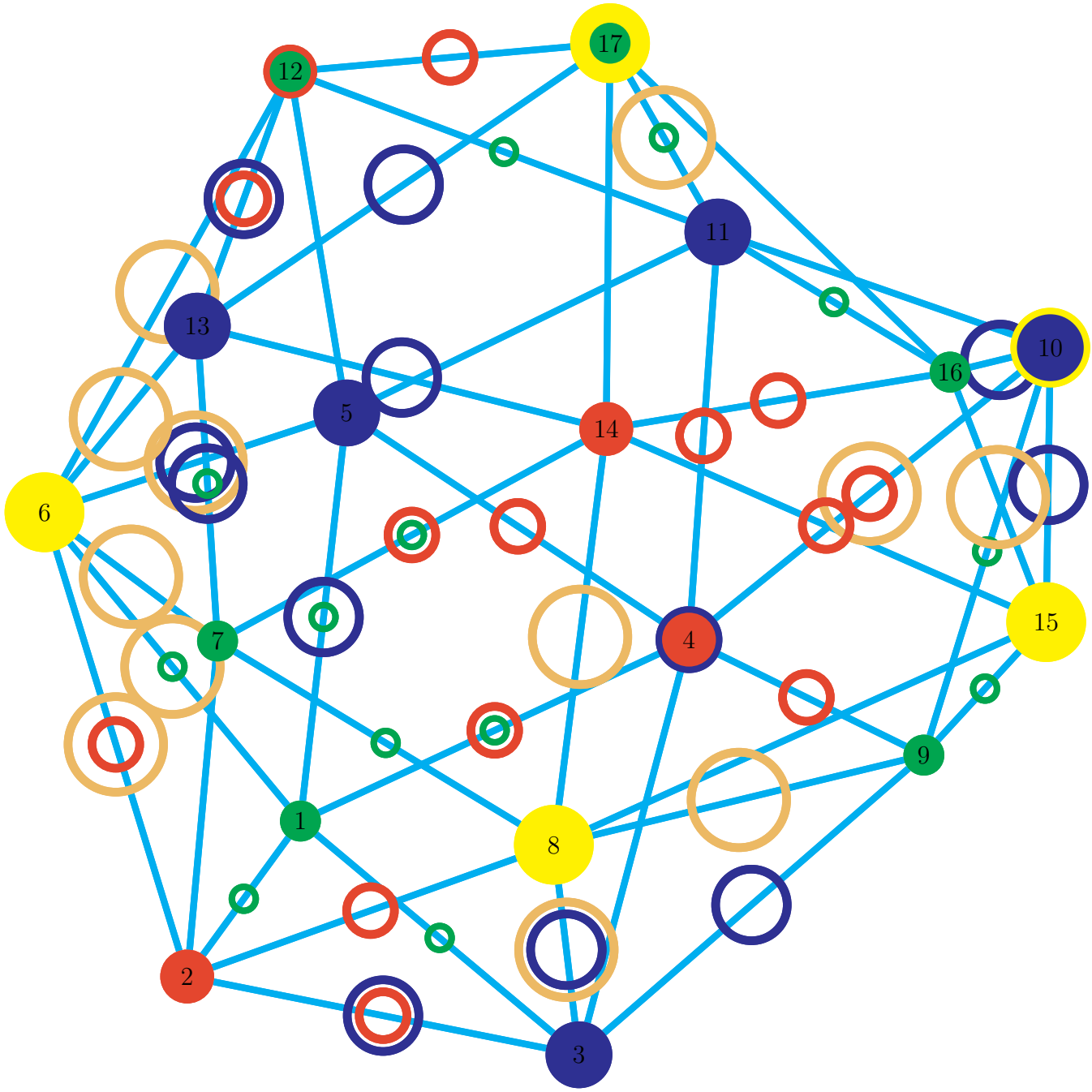

Figure 101: Instructions: 312: color edge 10→4 Yellow 313: uncolor vertex 4 Yellow

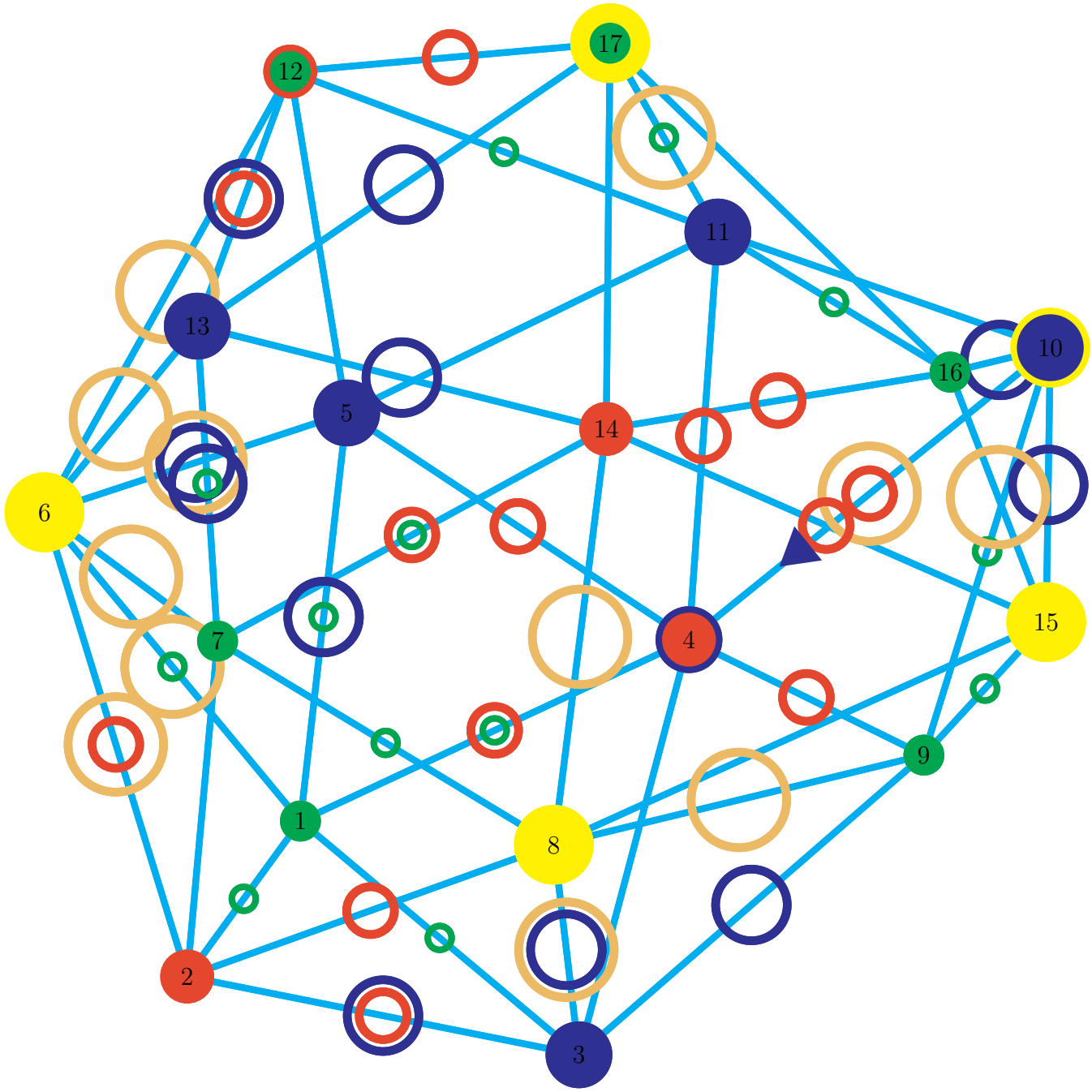

Figure 102: Instructions: 315: place edge 10→4 Blue ArrowR

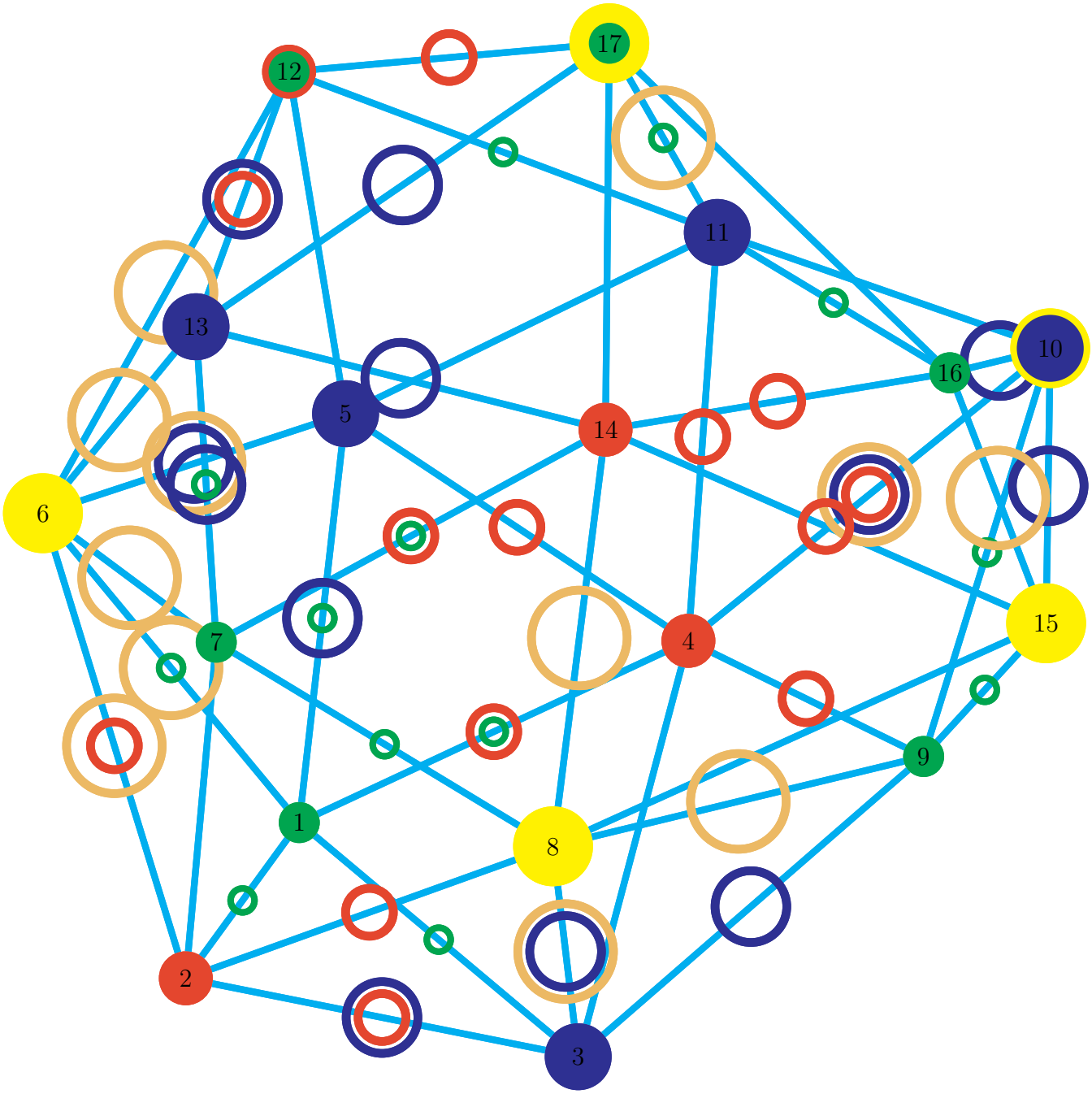

Figure 103: Instructions: 318: color edge 10→4 Blue 319: uncolor vertex 4 Blue

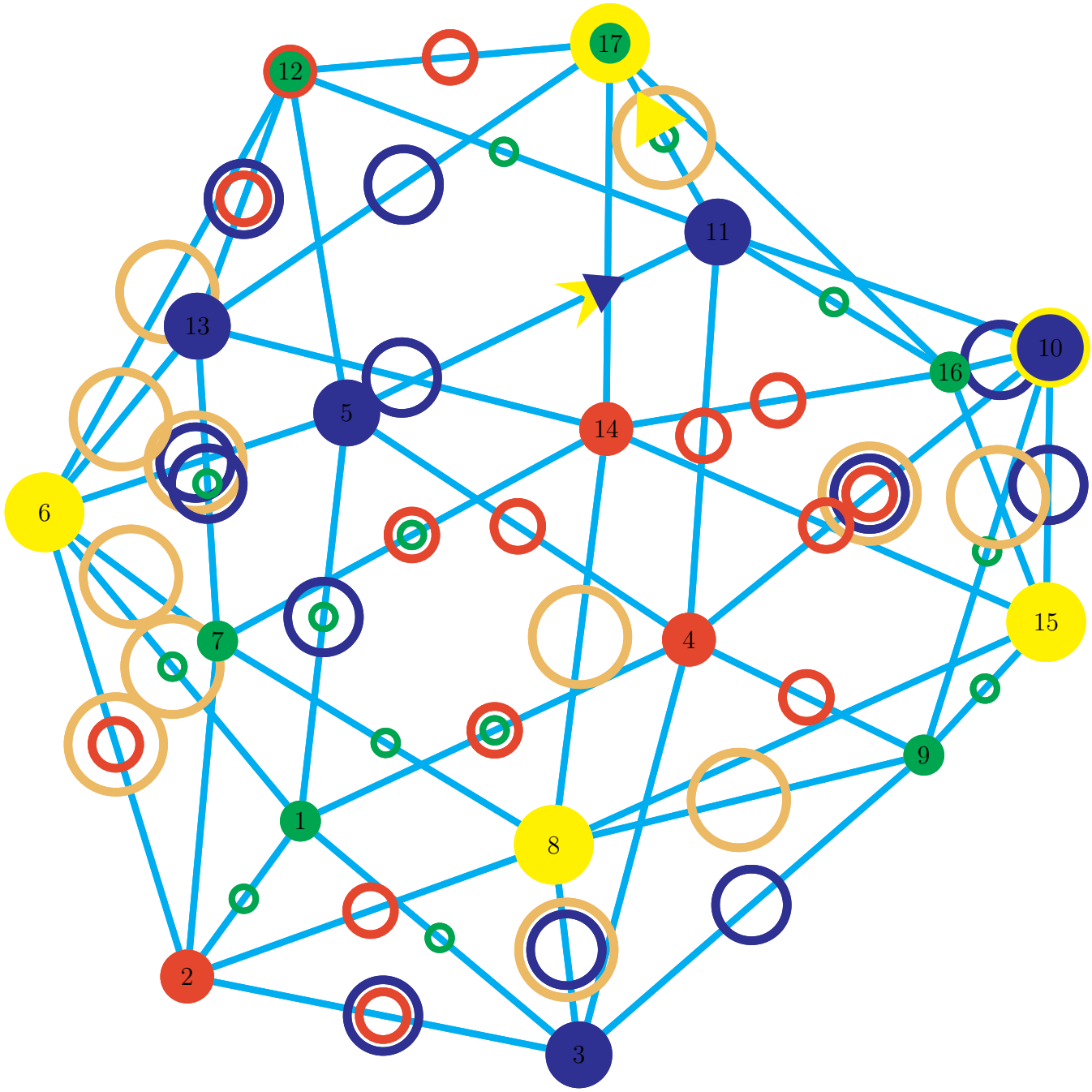

Figure 104: Instructions: 321: place edge 5→11 Blue ArrowR 322: place edge 5→11 Yellow ArrowI 323: place edge 11→17 Yellow ArrowR

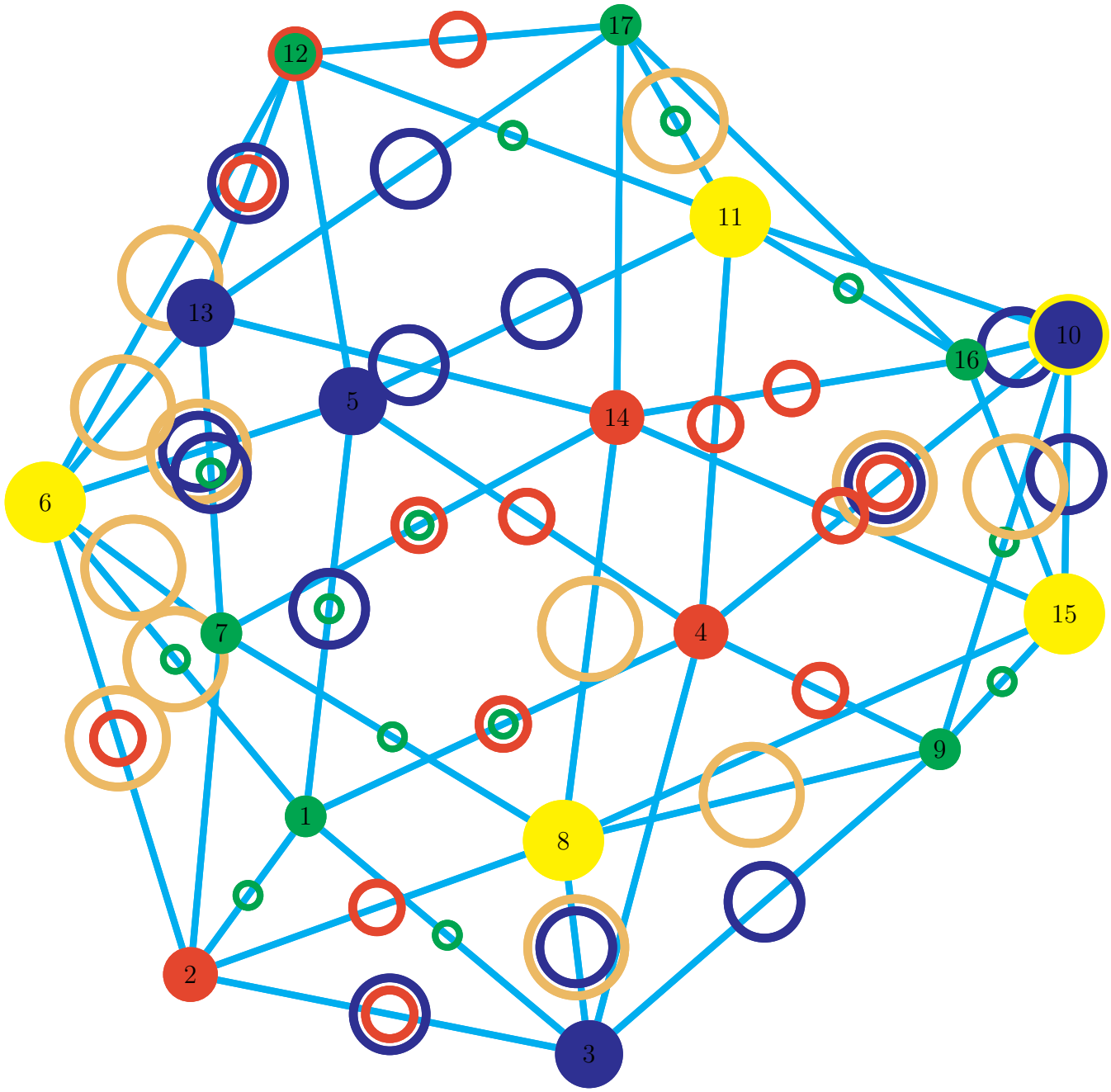

Figure 105: Instructions: 328: color edge 5→11 Blue 329: uncolor vertex 11 Blue 330: color vertex 11 Yellow 331: uncolor vertex 17 Yellow

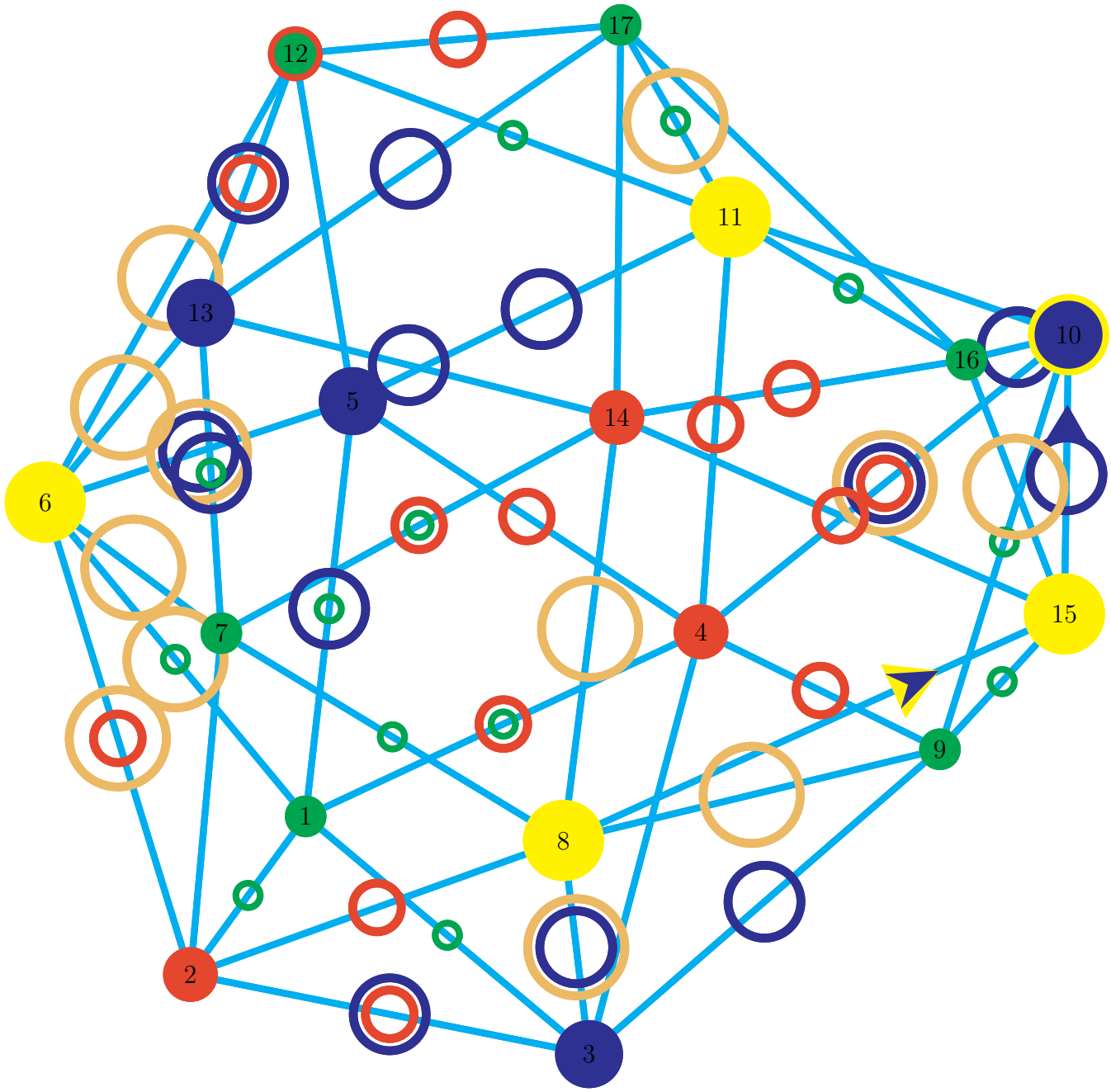

Figure 106: Instructions: 333: place edge 8→15 Yellow ArrowR 334: place edge 8→15 Blue ArrowI 335: place edge 15→10 Blue ArrowR

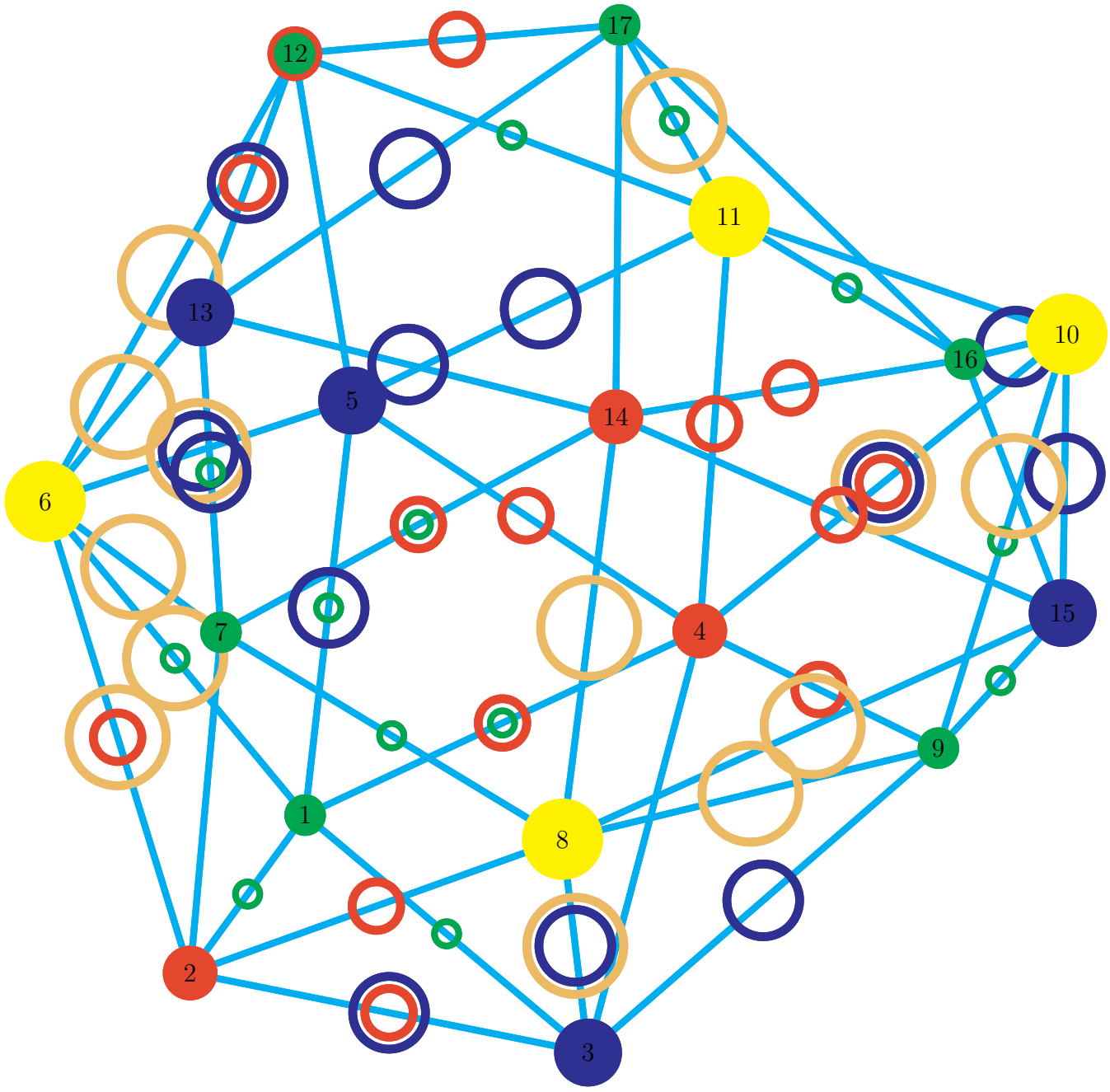

Figure 107: Instructions: 340: color edge 8→15 Yellow 341: uncolor vertex 15 Yellow 342: color vertex 15 Blue 343: uncolor vertex 10 Blue

Here property B is not satisfied for there is a blue circle on edge 10-16 but no blue checker on either of its vertices. The situation is remedied as follows.

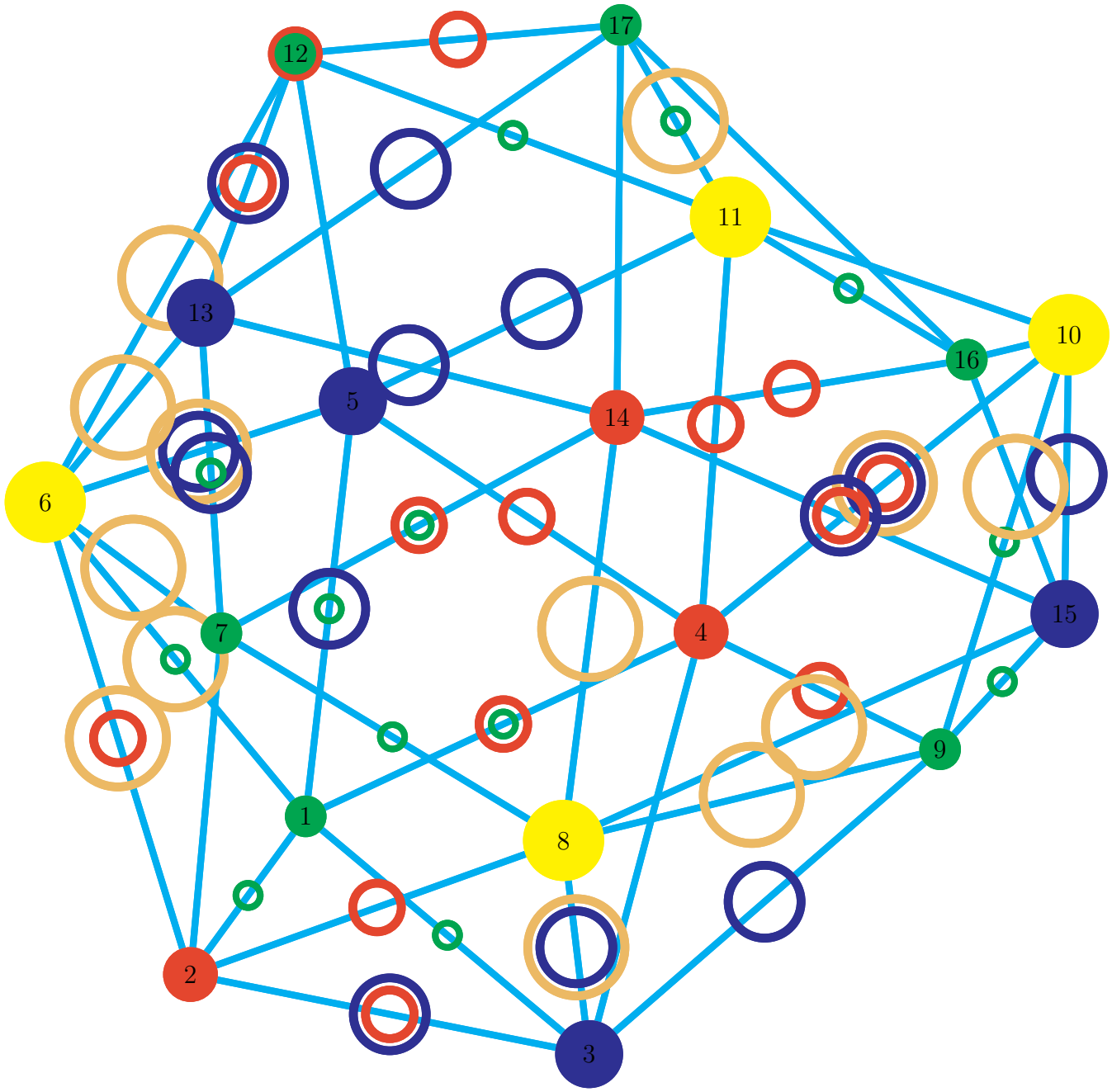

Figure 108: Instructions: 347: color edge 15→14 Blue

The blue circle on edge 10→16 is flipped to edge 14-15. Similary the Yellow circle on edge 15-16 is flipped to edge 11-16

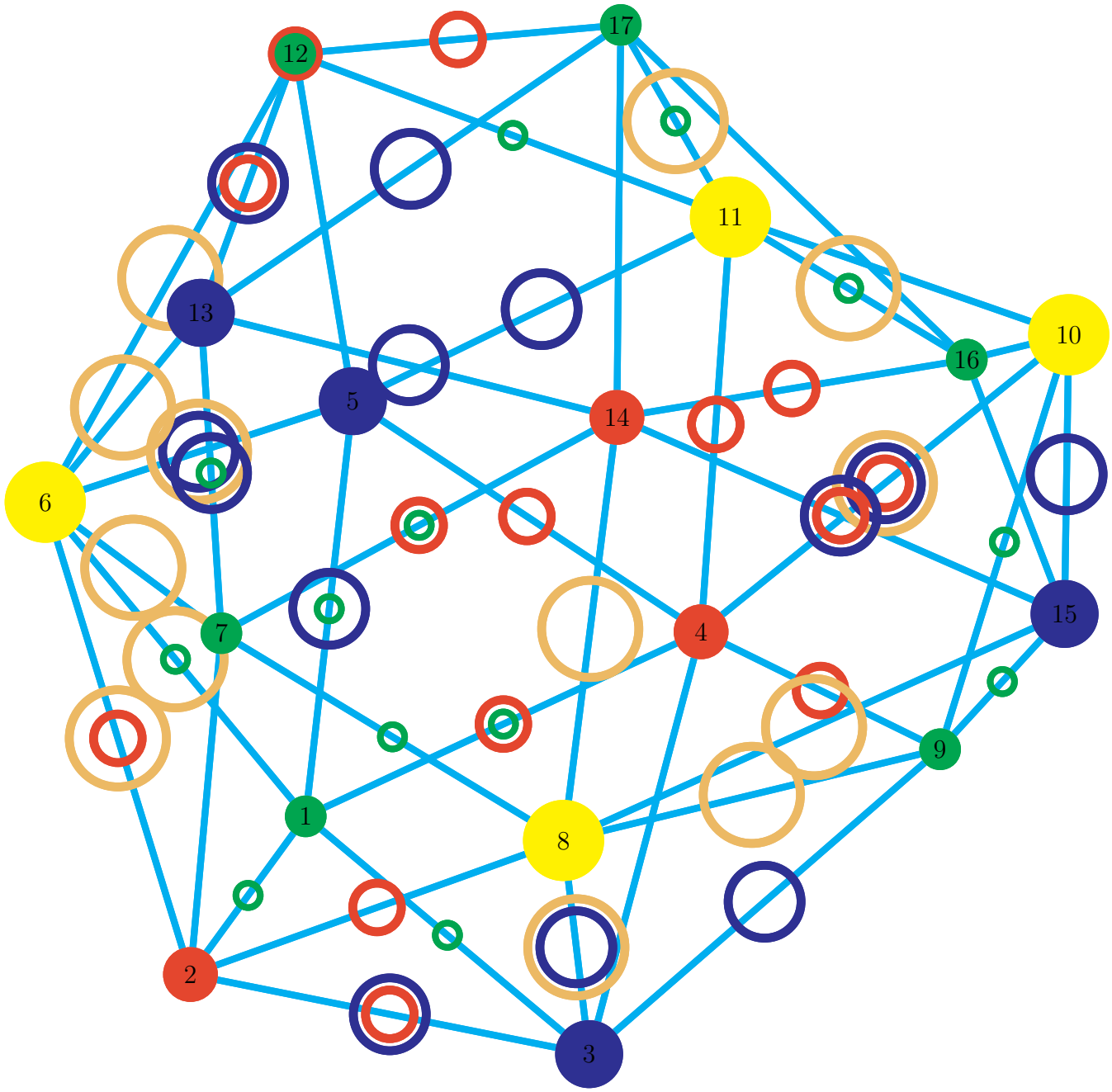

Figure 109: Instructions: 351: color edge 16→11 Yellow

Similarly the Blue circle on edge 10-4 is flipped to edge 4-5

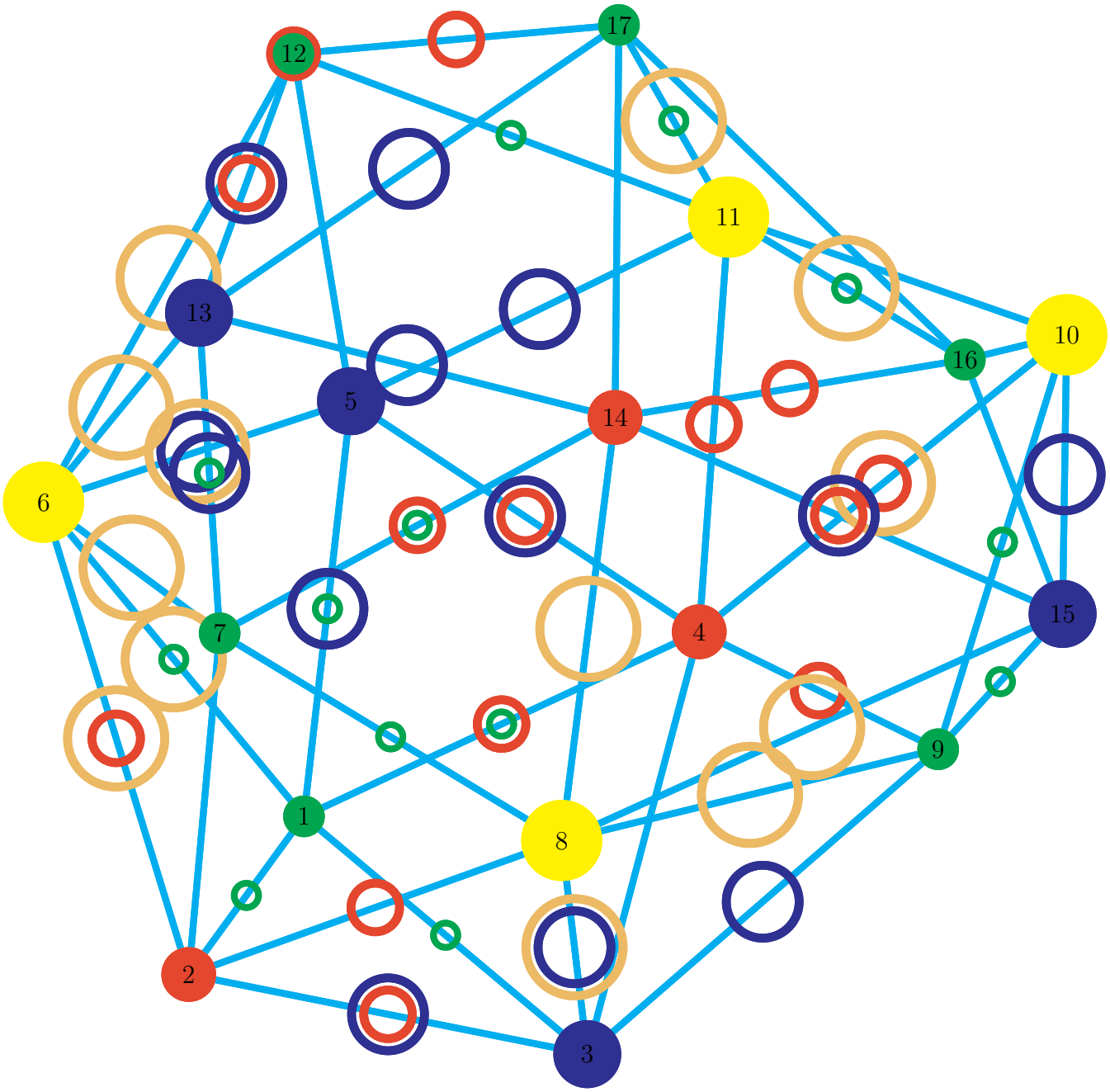

Figure 110: Instructions: 355: color edge 5→4 Blue

And property B is restored

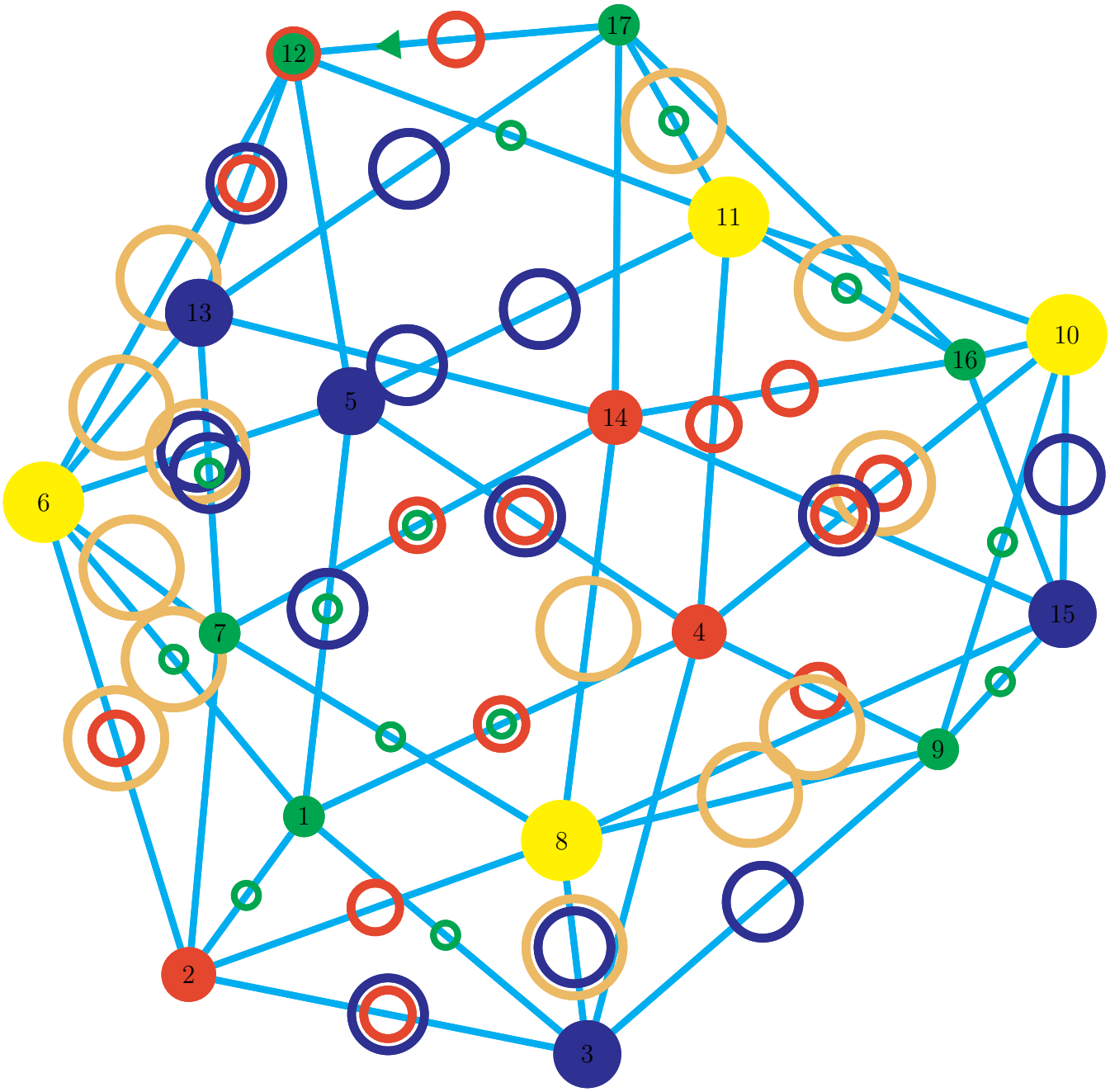

Figure 111: Instructions: 357: place edge 17→12 Green ArrowR

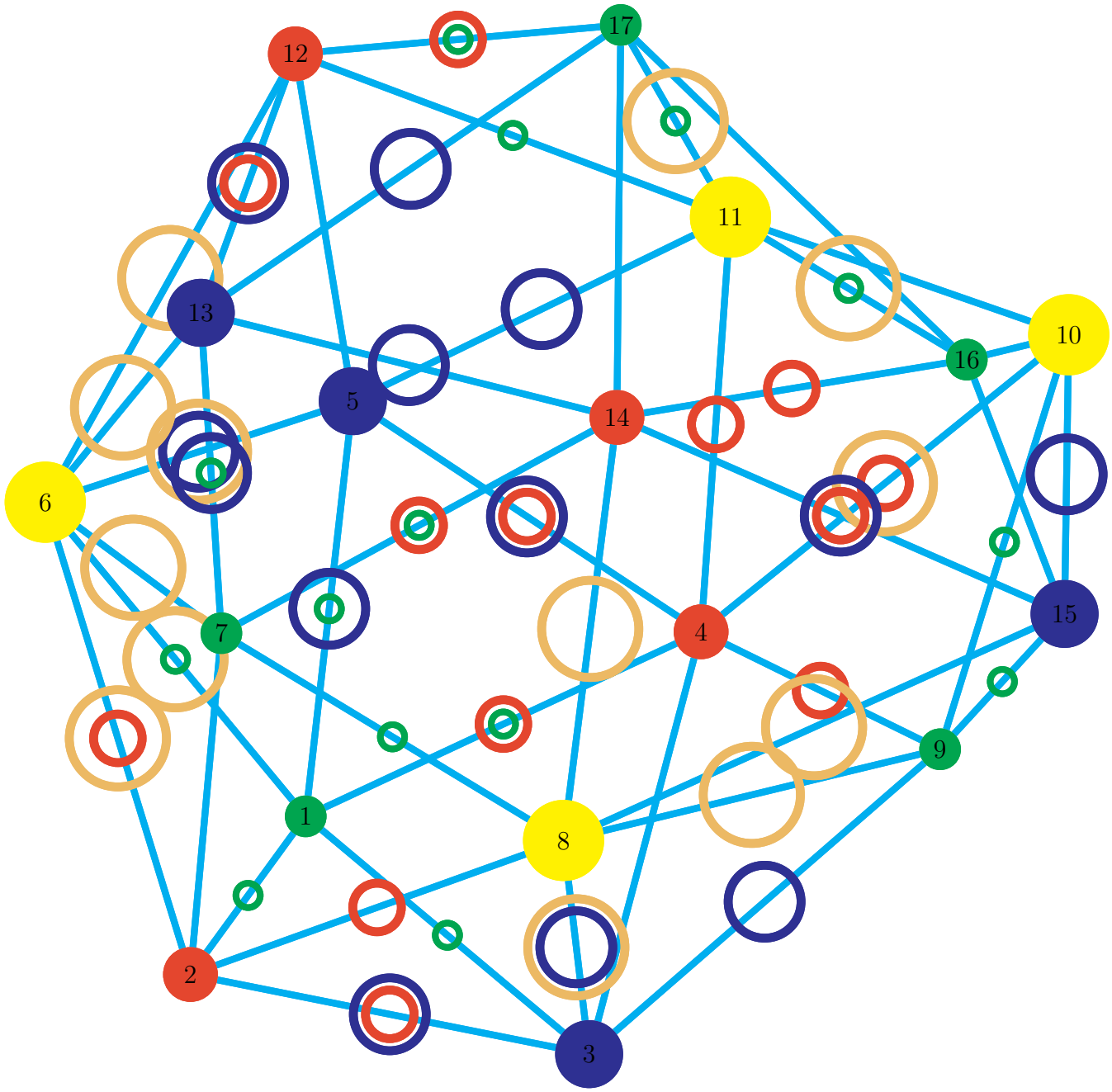

Figure 112: Instructions: 360: color edge 17→12 Green 361: uncolor vertex 12 Green

After executing the previous instruction property B fails on edge 11-12, however it is restored by moving the green circle on edge 11-12 to edge 14-17

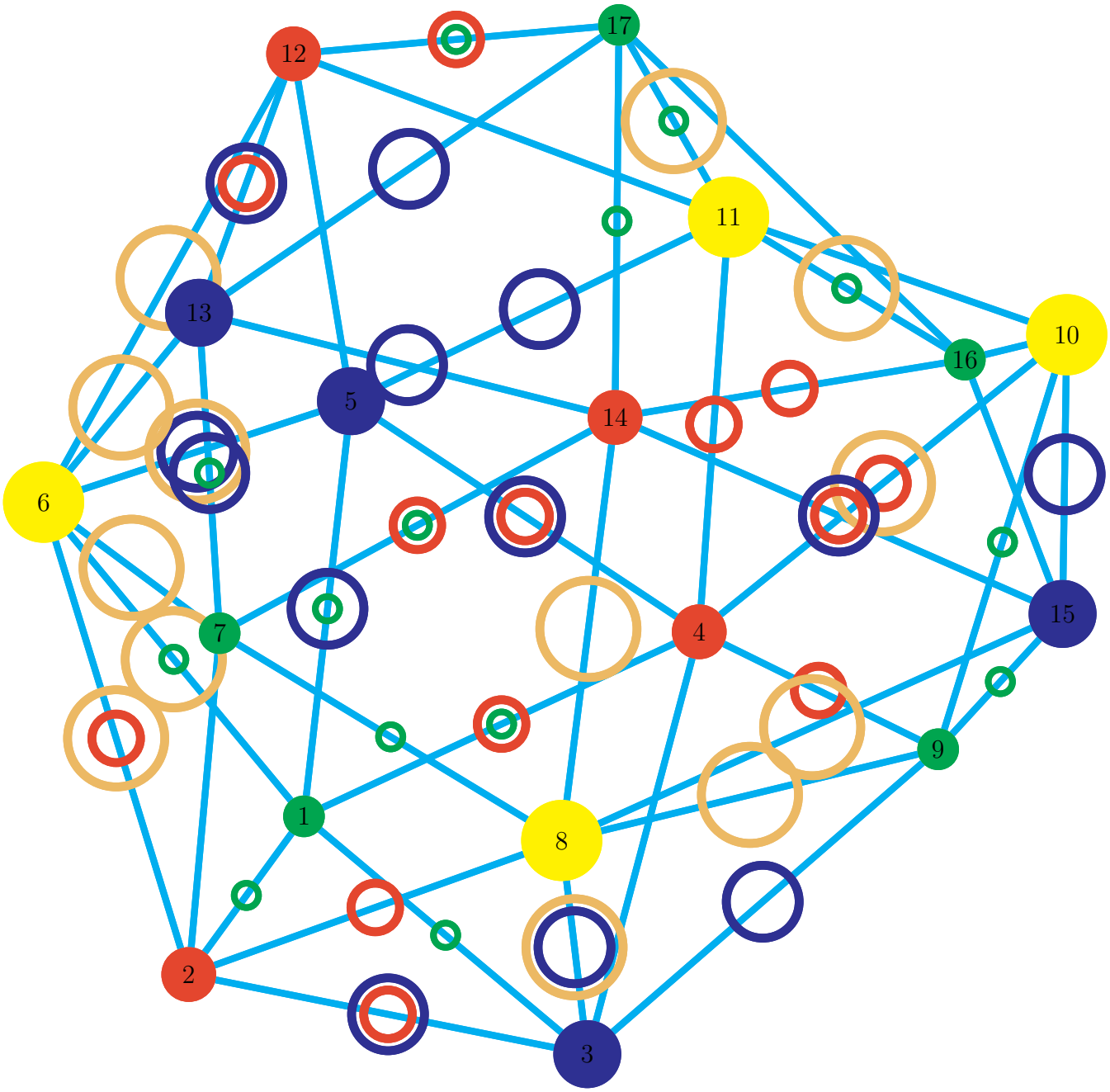

Figure 113: Instructions: 365: color edge 14→17 Green

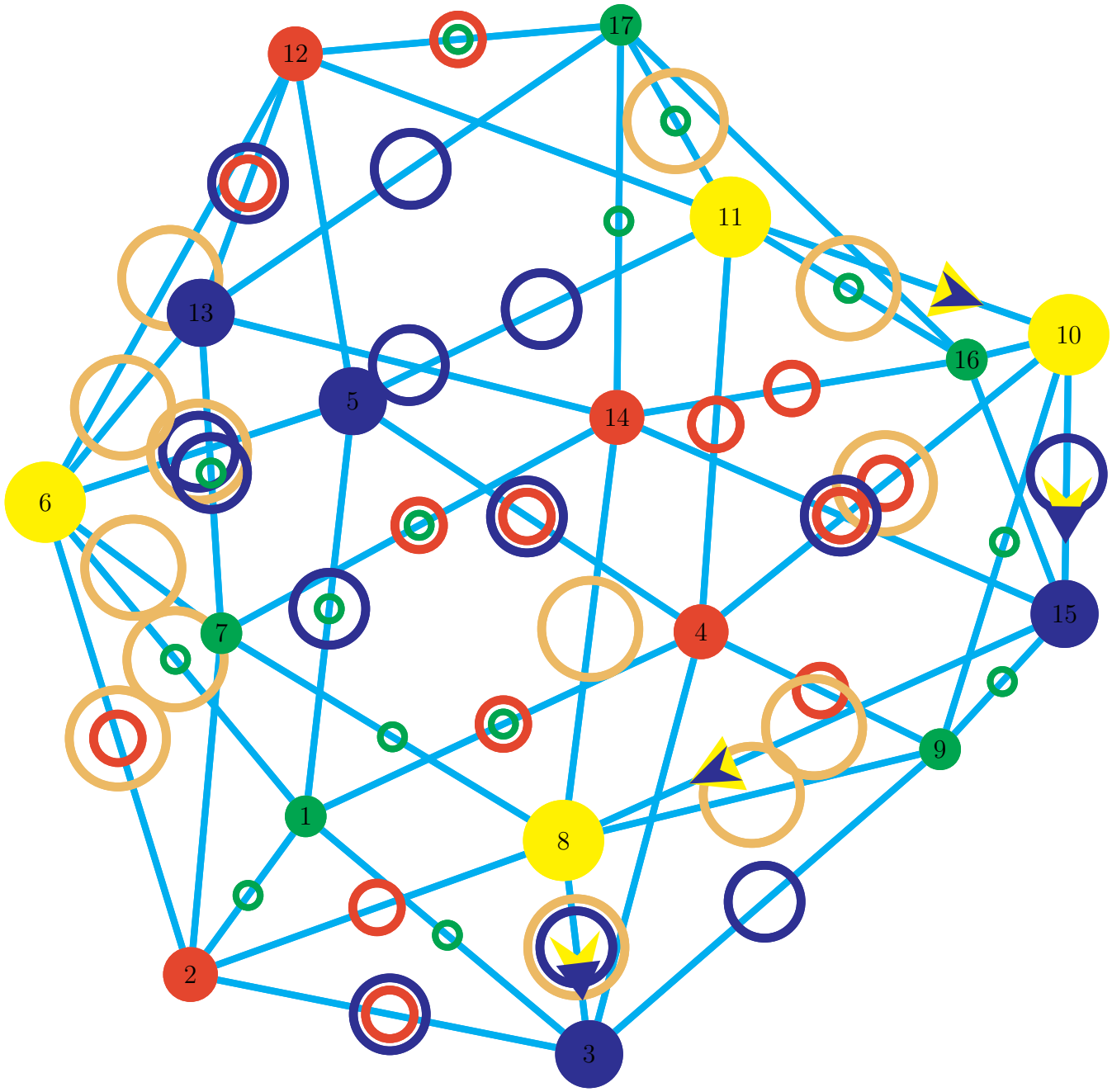

Figure 114: Instructions: 367: place edge 11→10 Yellow ArrowR 368: place edge 11→10 Blue ArrowI 369: place edge 10→15 Blue ArrowR 370: place edge 10→15 Yellow ArrowI 371: place edge 15→8 Blue ArrowI 372: place edge 15→8 Yellow ArrowR 373: place edge 8→3 Blue ArrowR 374: place edge 8→3 Yellow ArrowI

Execution of this alternating chain will cause property B to fail, but it is restored in the usual manner

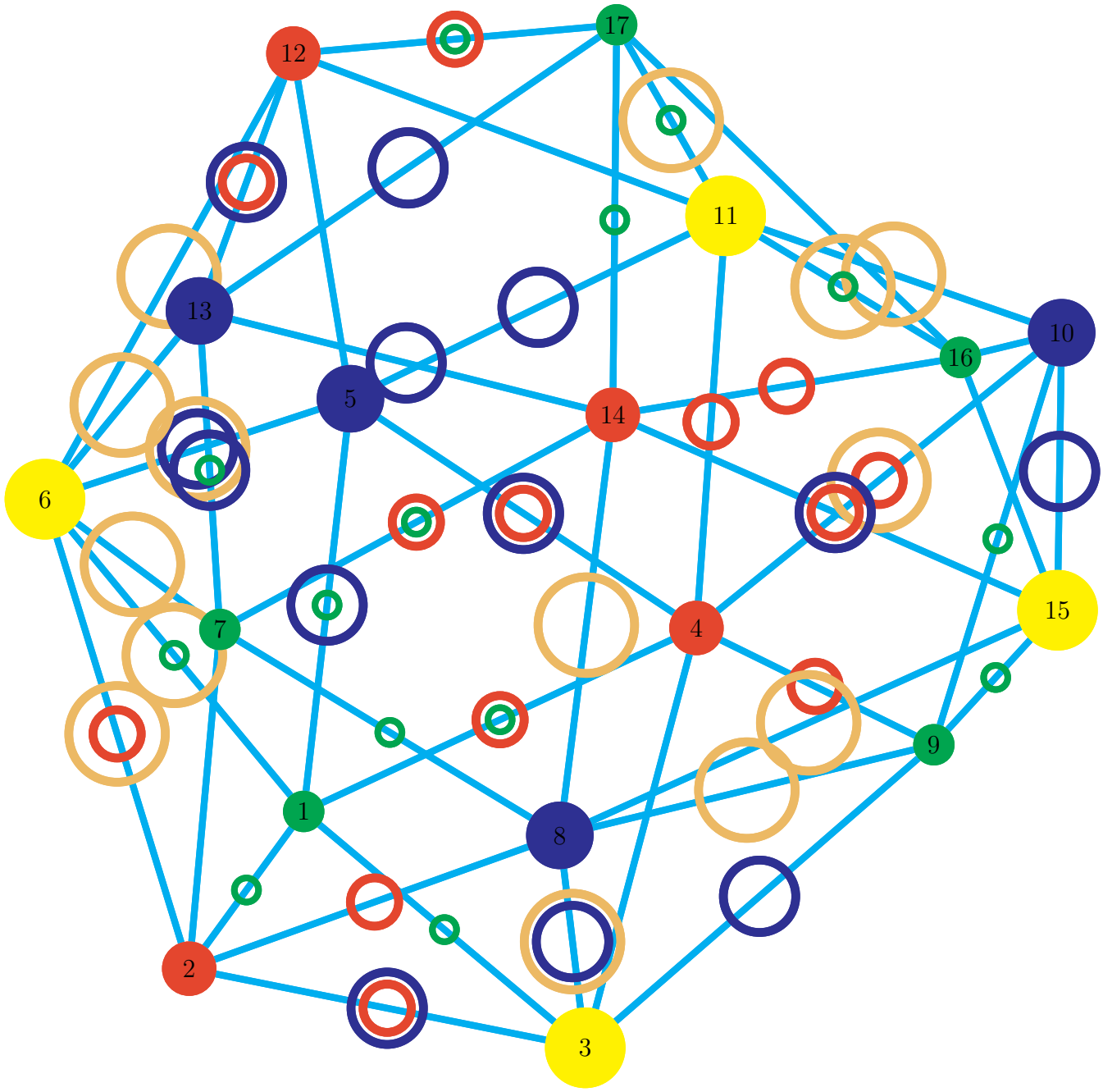

Figure 115: Instructions: 384: color edge 11→10 Yellow 385: uncolor vertex 10 Yellow 386: color vertex 10 Blue 387: uncolor vertex 15 Blue 388: color vertex 15 Yellow 389: uncolor vertex 8 Yellow 390: color vertex 8 Blue 391: uncolor vertex 3 Blue 392: color vertex 3 Yellow

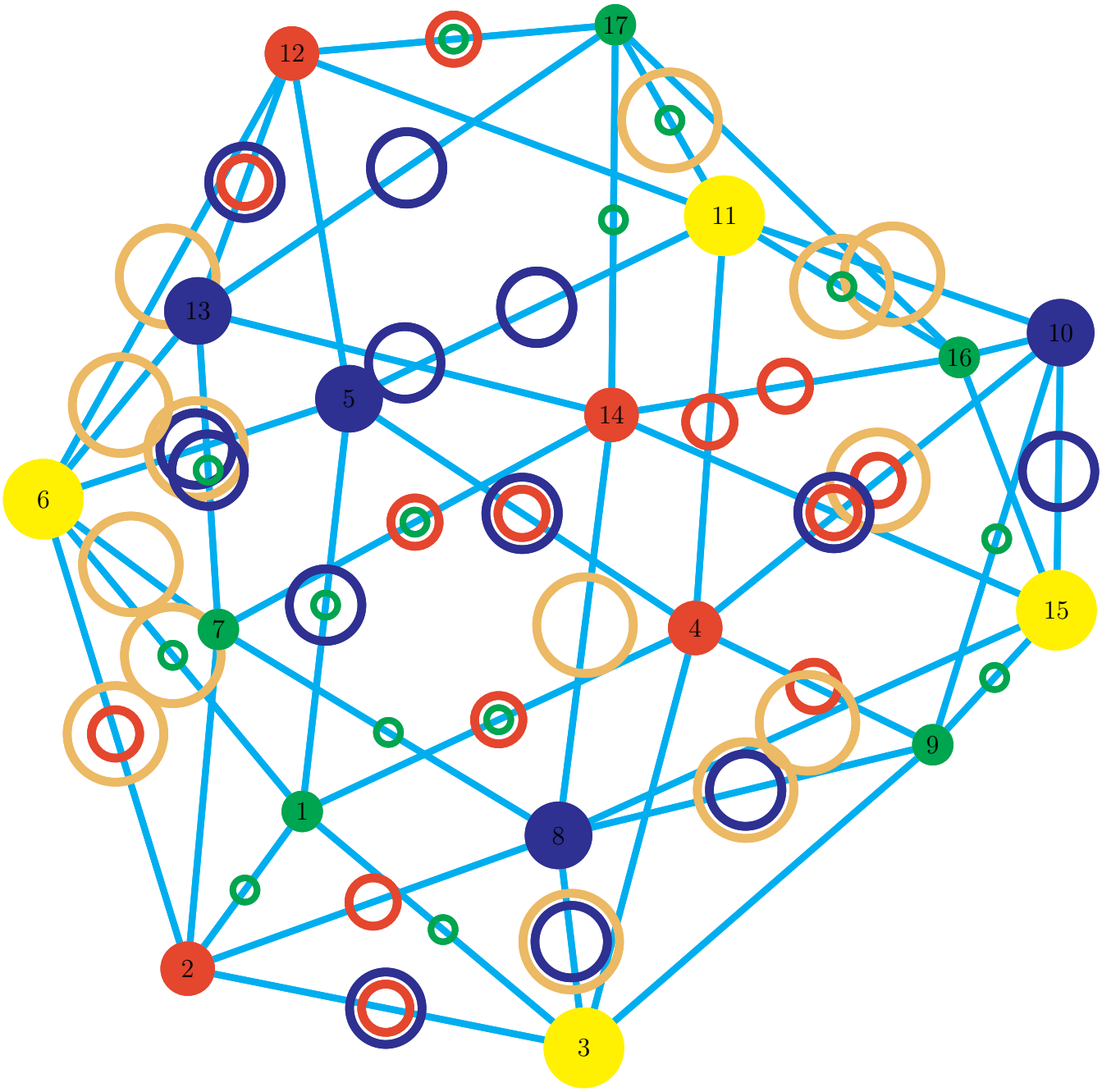

Figure 116: Instructions: 396: color edge 9→8 Blue

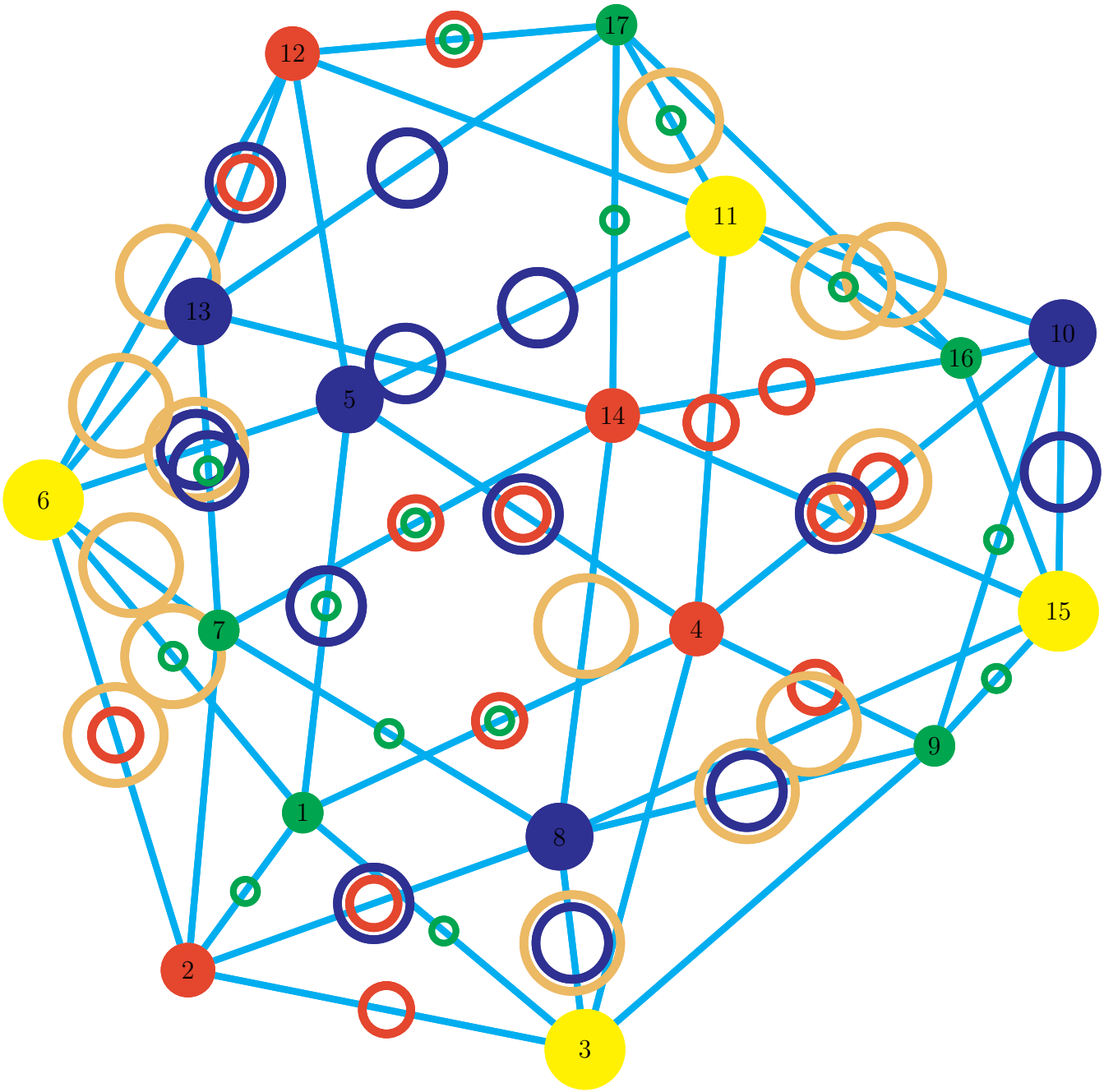

Figure 117: Instructions: 400: color edge 2→8 Blue

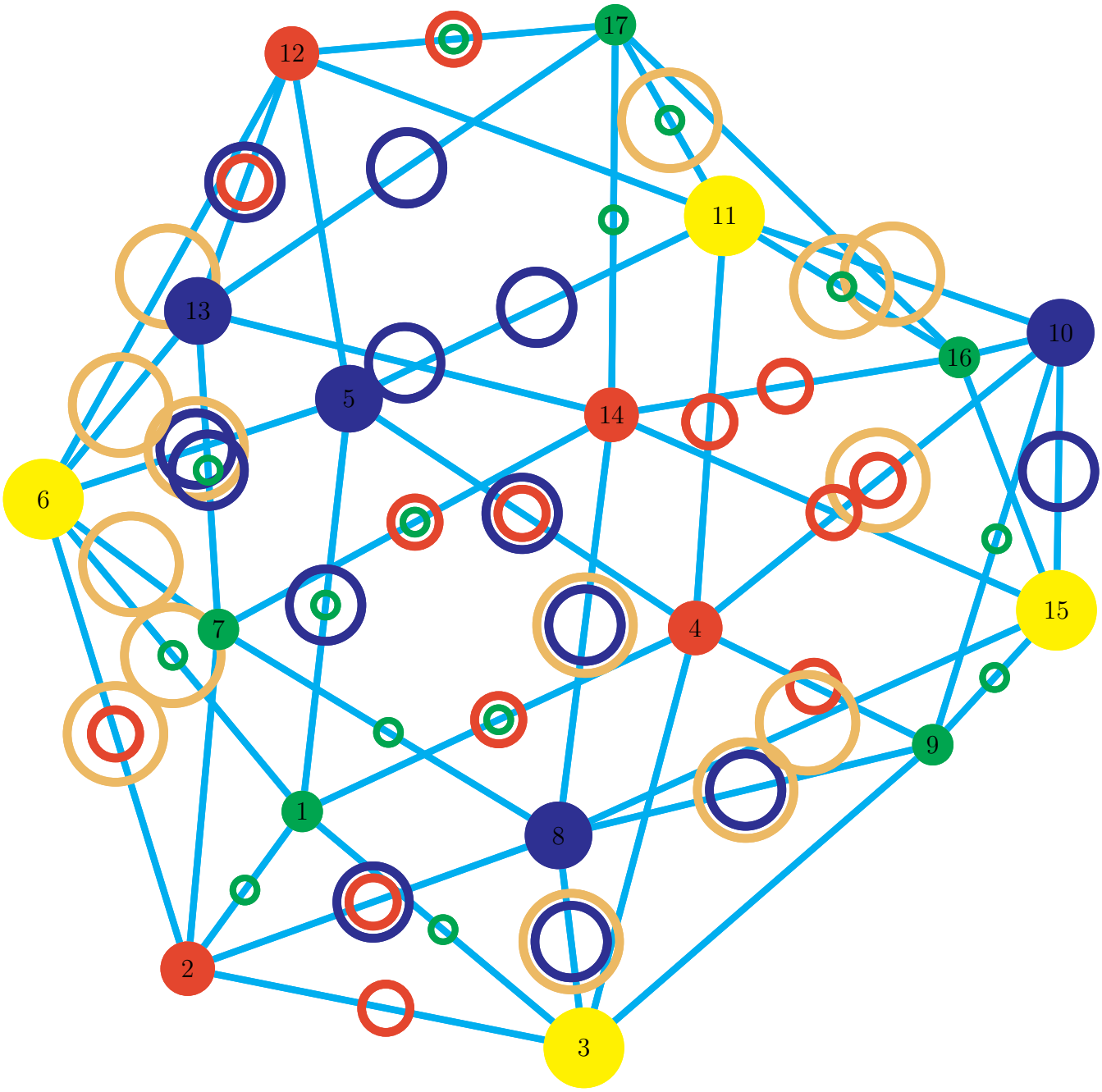

Figure 118: Instructions: 404: color edge 8→14 Blue

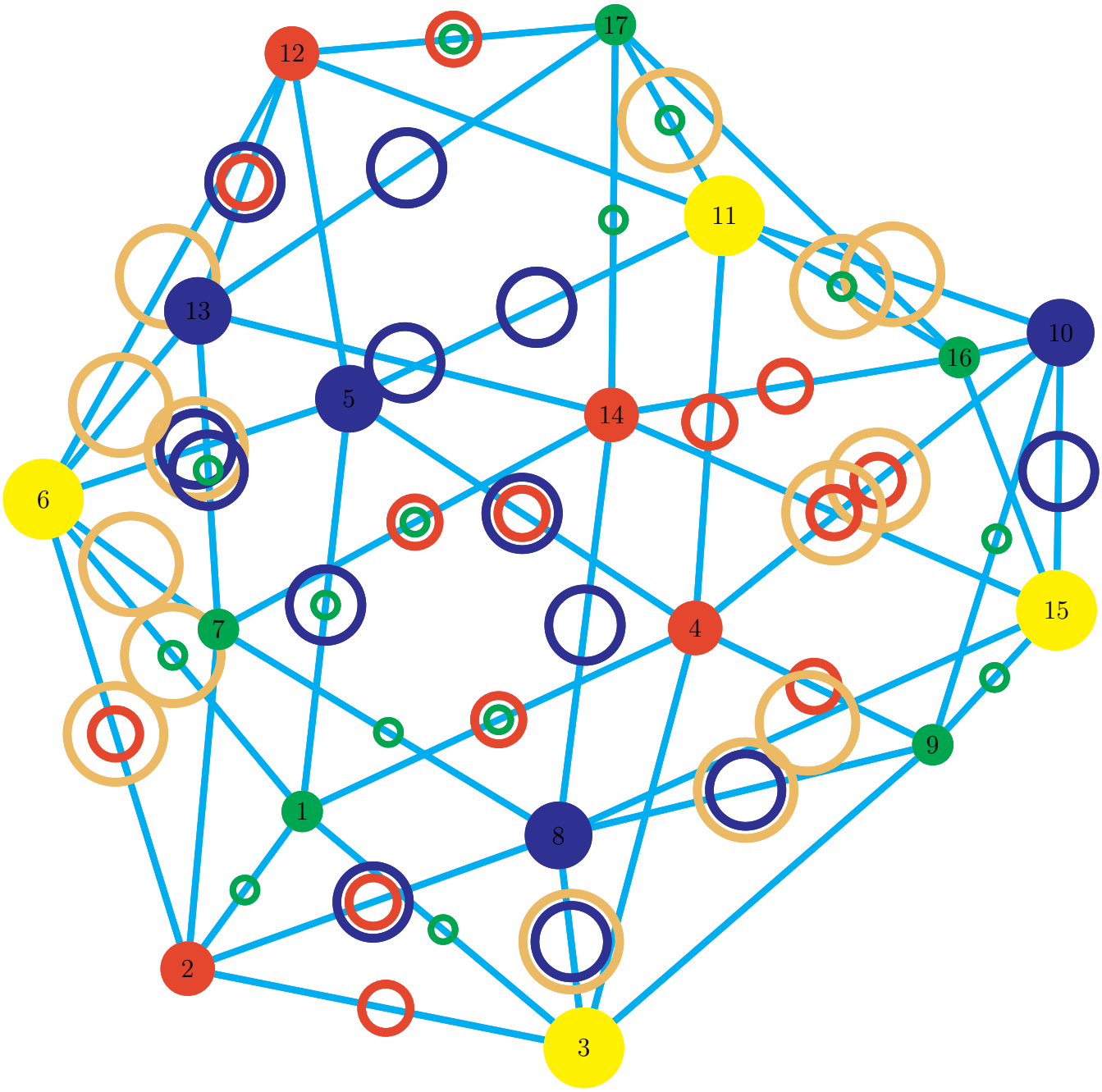

Figure 119: Instructions: 408: color edge 15→14 Yellow

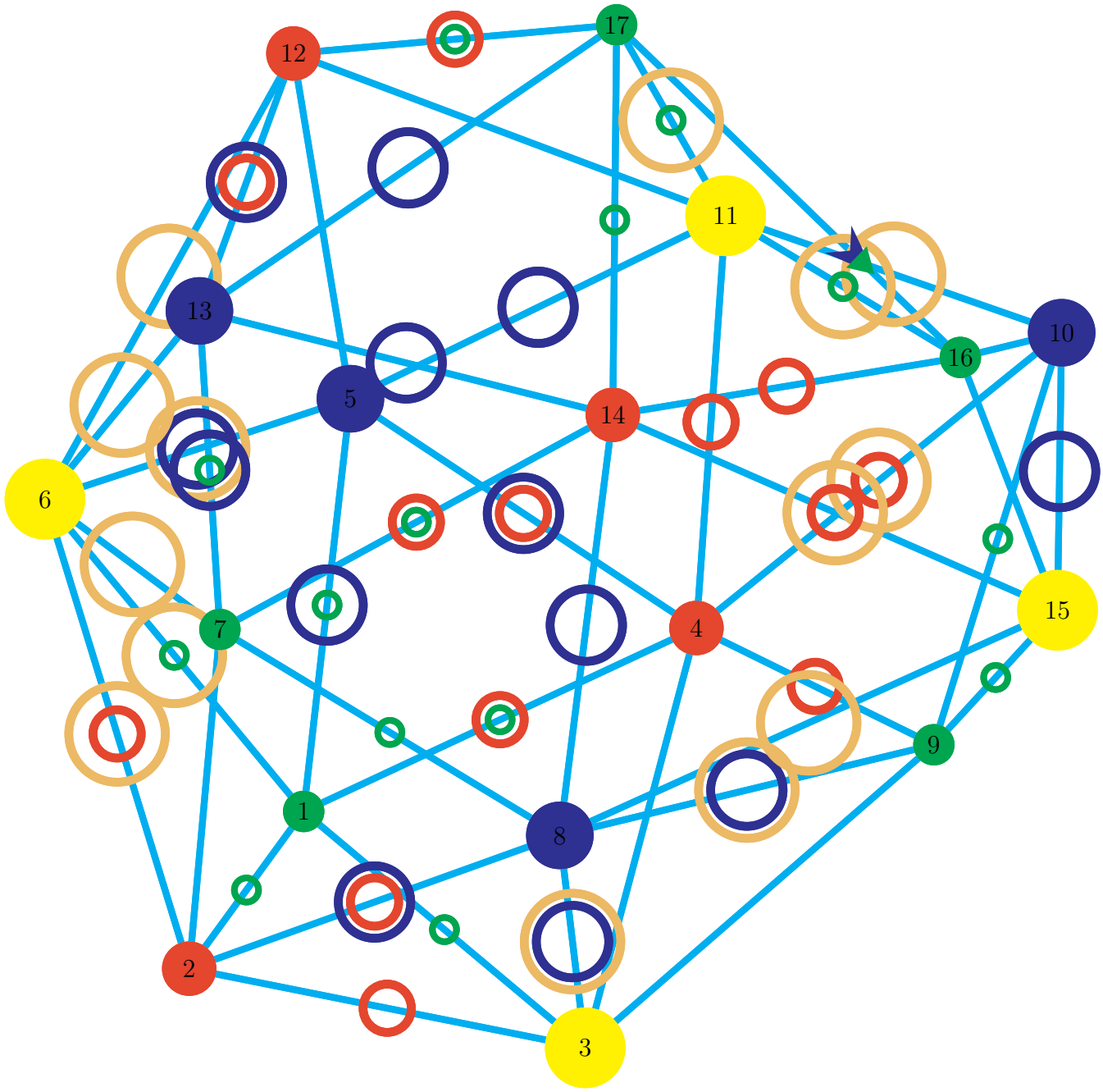

Figure 120: Instructions: 410: place edge 17→16 Green ArrowR 411: place edge 17→16 Blue ArrowI

Execution of this alternating chain will cause property B to fail, but it is restored in the usual manner

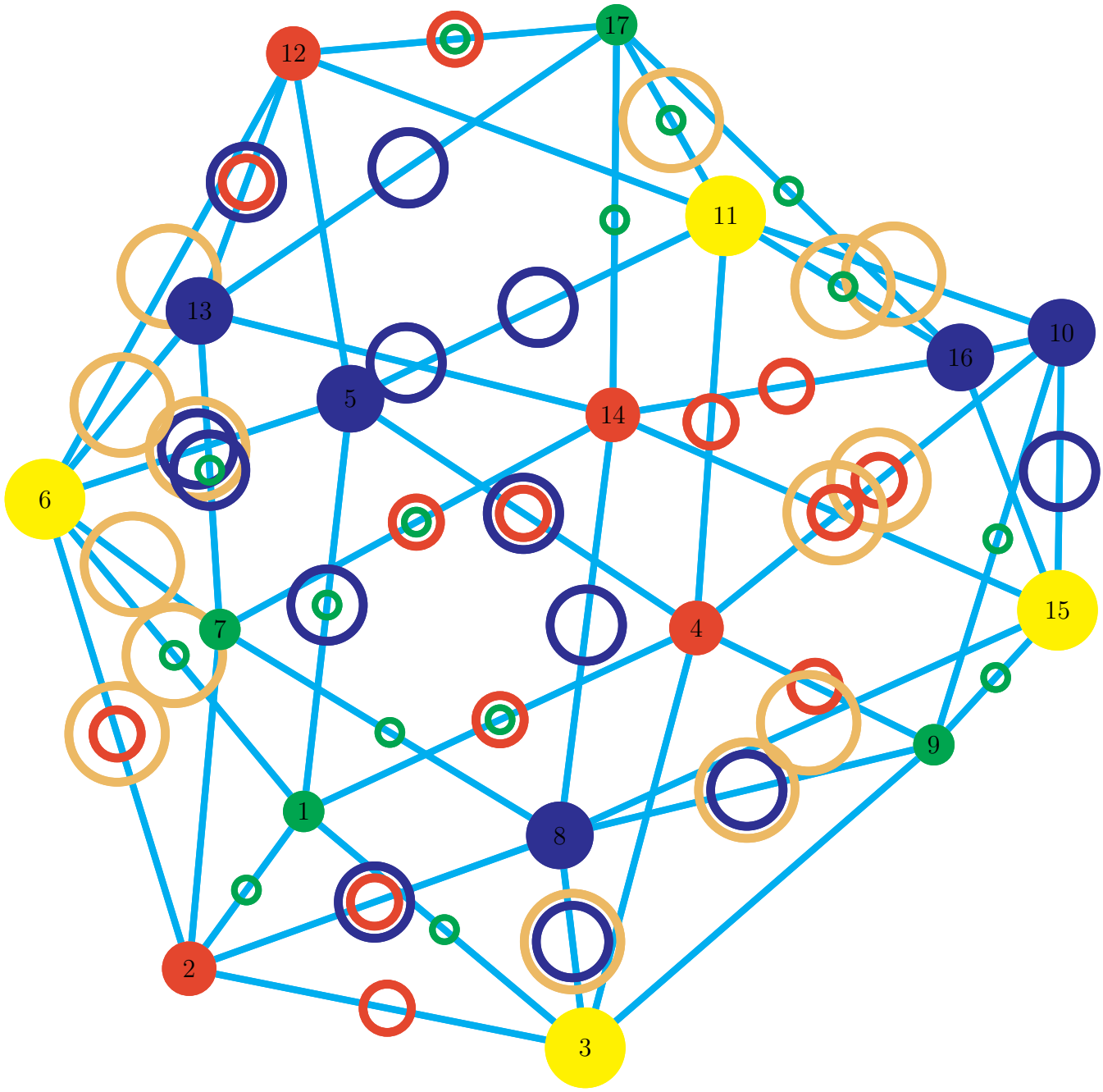

Figure 121: Instructions: 415: color edge 17→16 Green 416: uncolor vertex 16 Green 417: color vertex 16 Blue

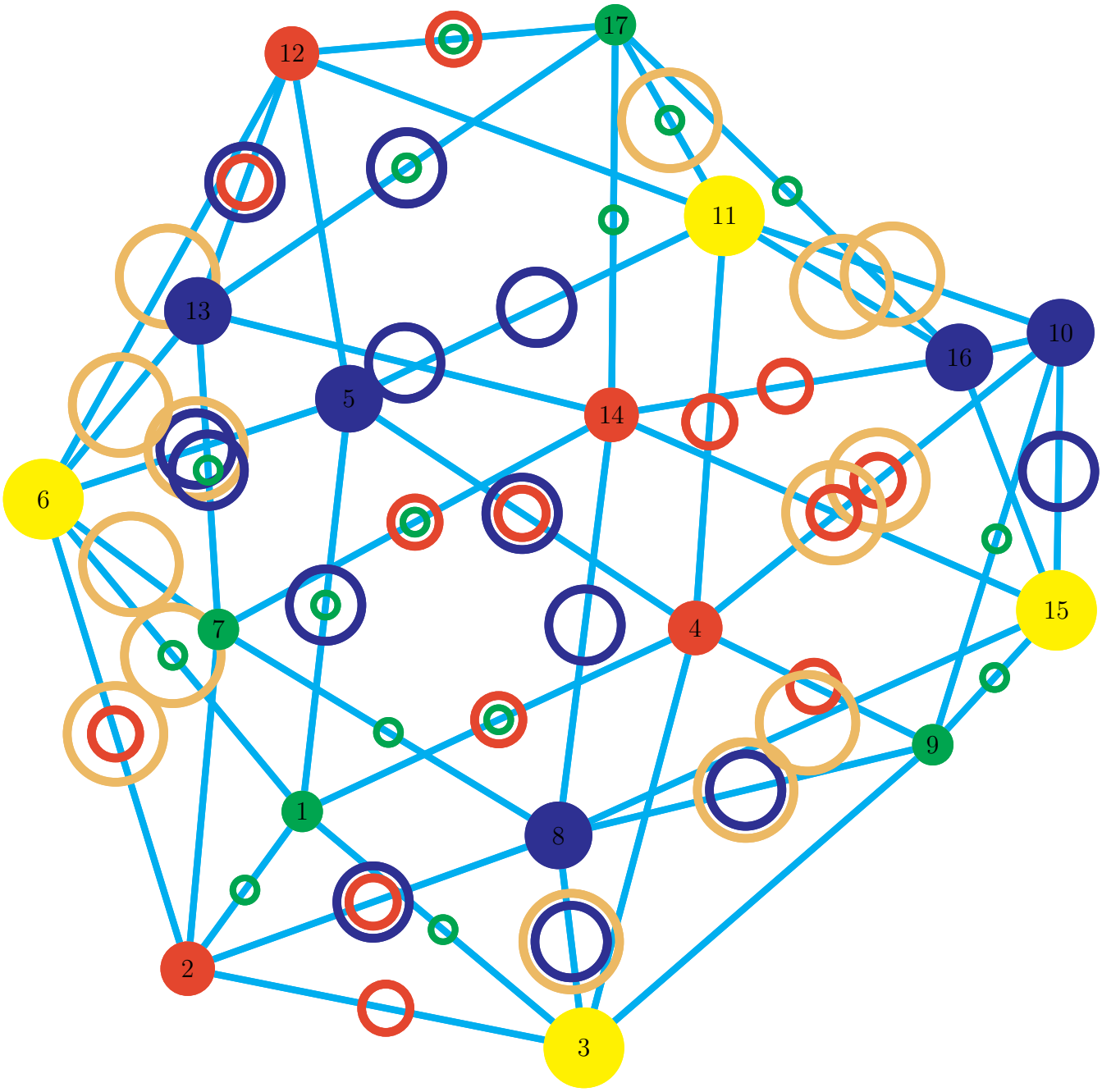

Figure 122: Instructions: 421: color edge 17→13 Green

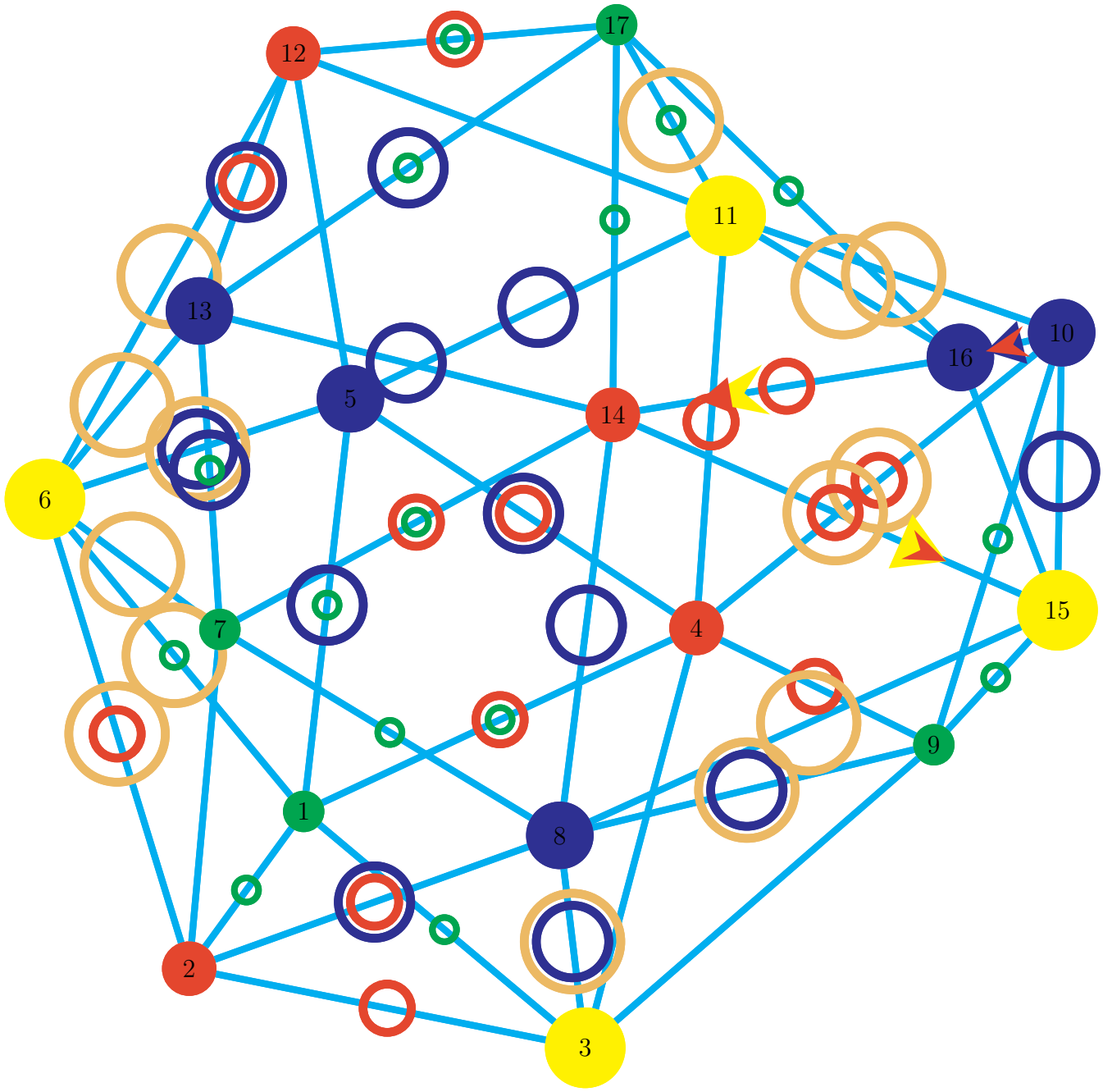

Figure 123: Instructions: 423: place edge 10→16 Blue ArrowR 424: place edge 10→16 Red ArrowI 425: place edge 16→14 Red ArrowR 426: place edge 16→14 Yellow ArrowI 427: place edge 14→15 Red ArrowI 428: place edge 14→15 Yellow ArrowR

Execution of this alternating chain will cause property B to fail, but it is restored in the usual manner

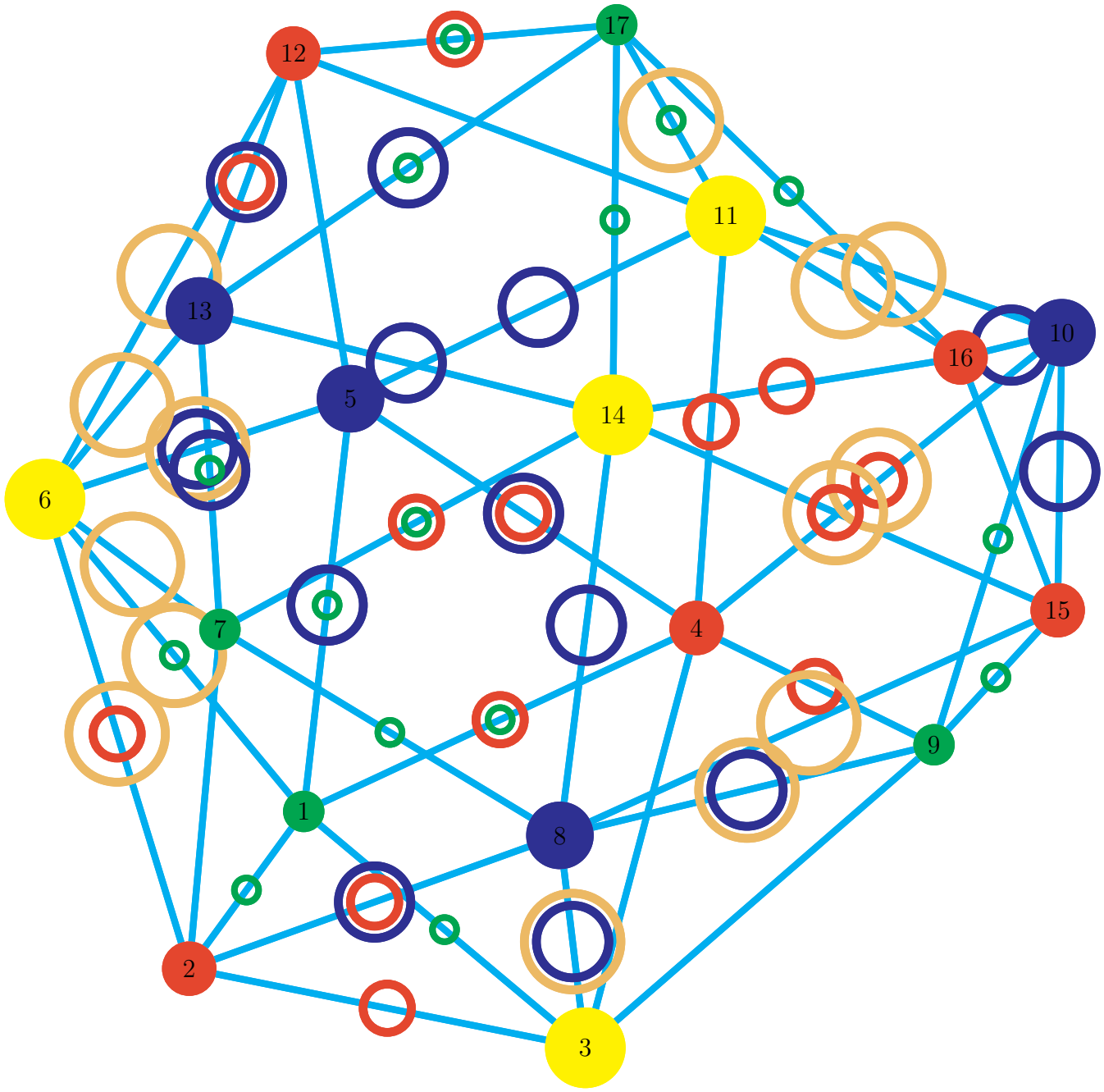

Figure 124: Instructions: 436: color edge 10→16 Blue 437: uncolor vertex 16 Blue 438: color vertex 16 Red 439: uncolor vertex 14 Red 440: color vertex 14 Yellow 441: uncolor vertex 15 Yellow 442: color vertex 15 Red

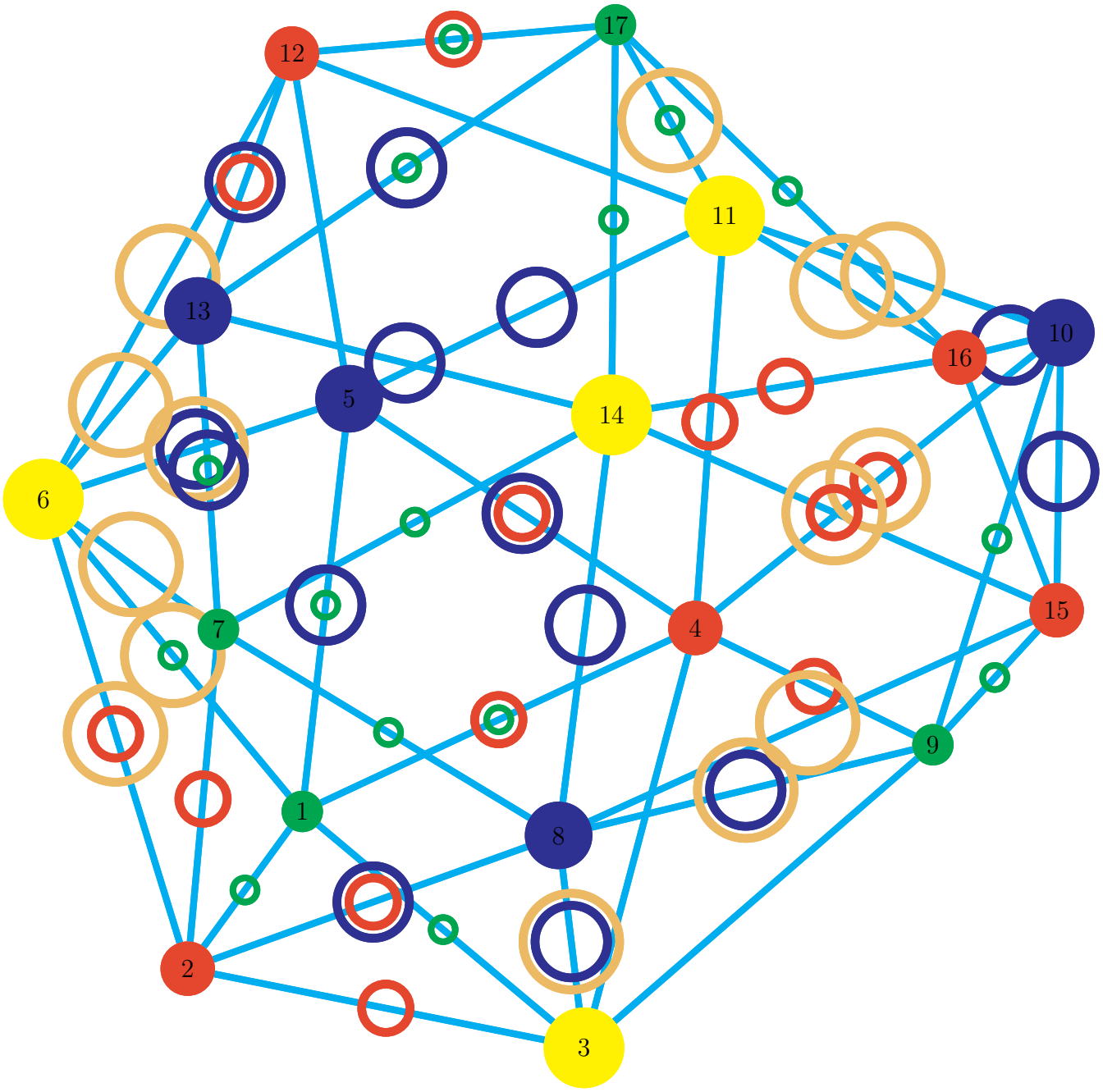

Figure 125: Instructions: 446: color edge 7→2 Red

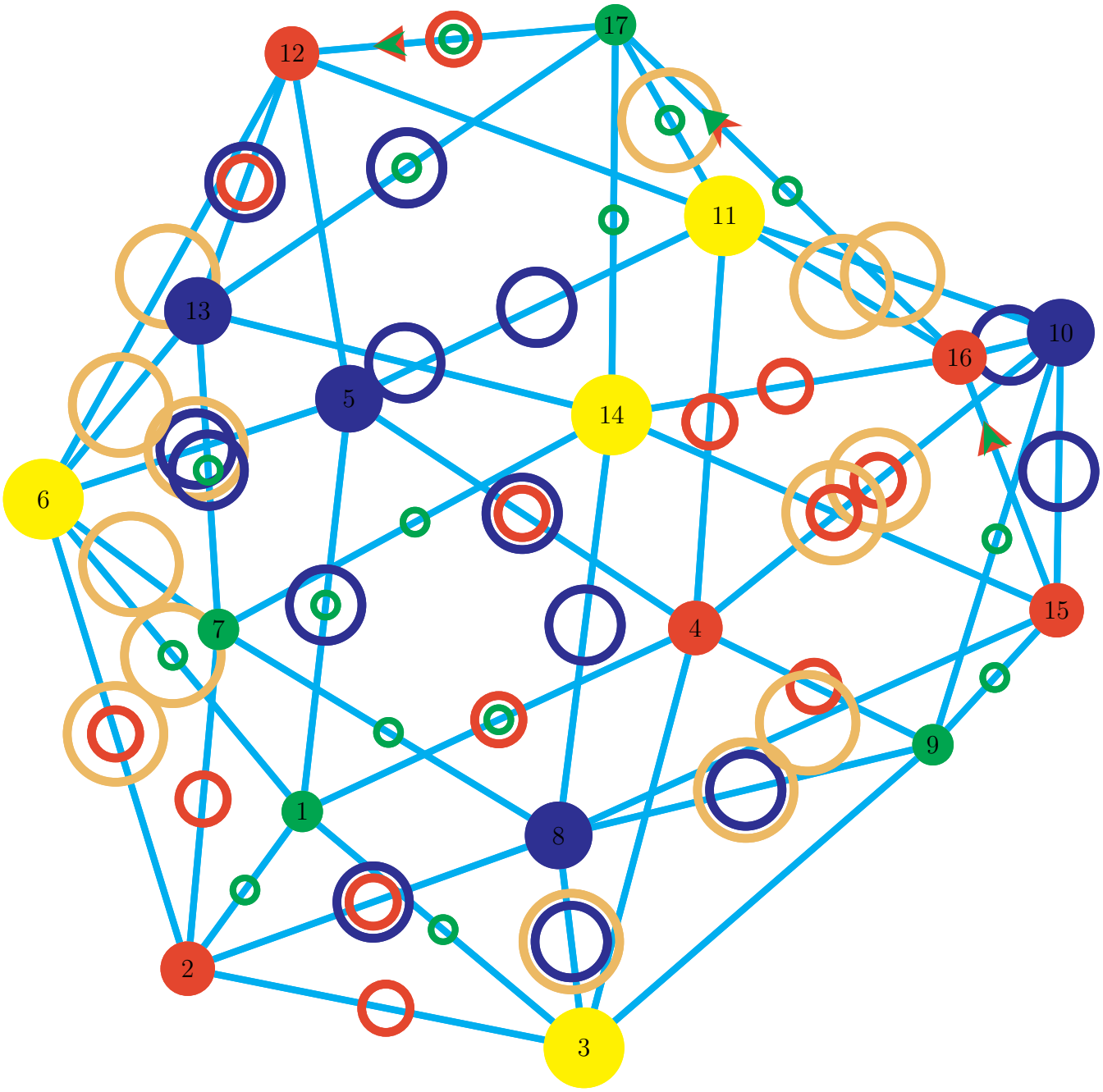

Figure 126: Instructions: 448: place edge 15→16 Red ArrowR 449: place edge 15→16 Green ArrowI 450: place edge 16→17 Green ArrowR 451: place edge 16→17 Red ArrowI 452: place edge 17→12 Green ArrowI 453: place edge 17→12 Red ArrowR

Execution of this alternating chain will cause property B to fail, but it is restored in the usual manner

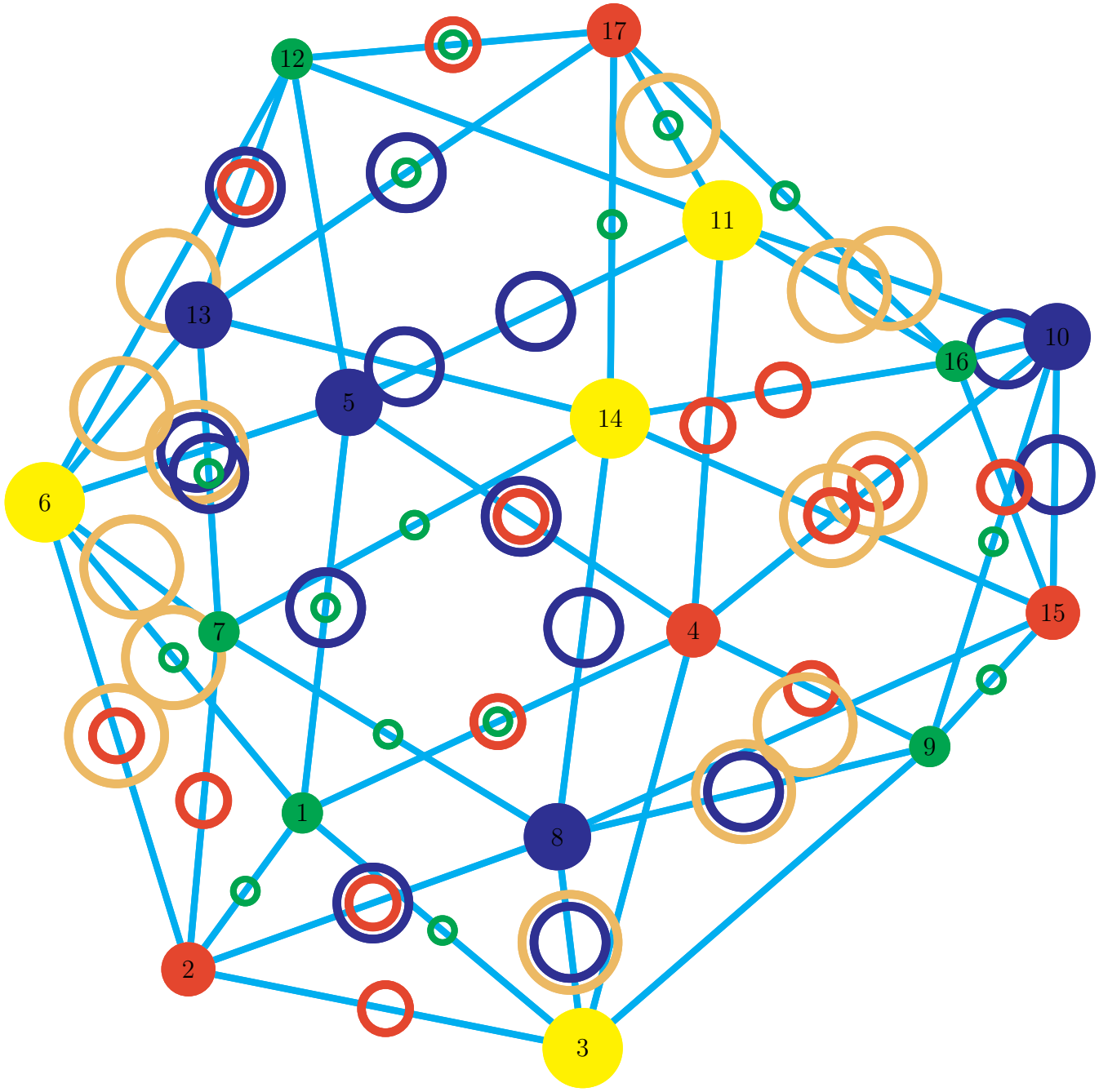

Figure 127: Instructions: 461: color edge 15→16 Red 462: uncolor vertex 16 Red 463: color vertex 16 Green 464: uncolor vertex 17 Green 465: color vertex 17 Red 466: uncolor vertex 12 Red 467: color vertex 12 Green

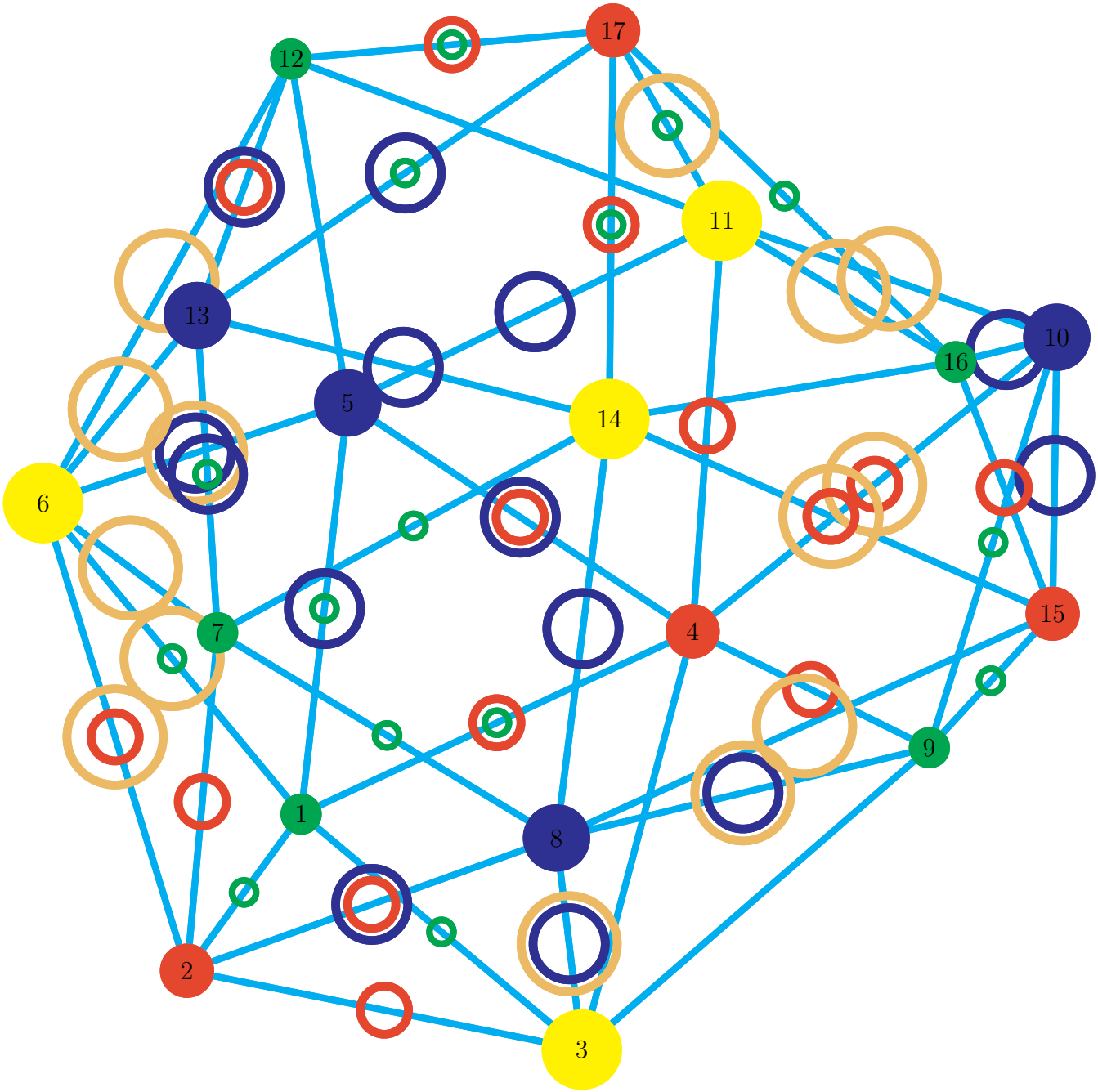

Figure 128: Instructions: 471: color edge 17→14 Red

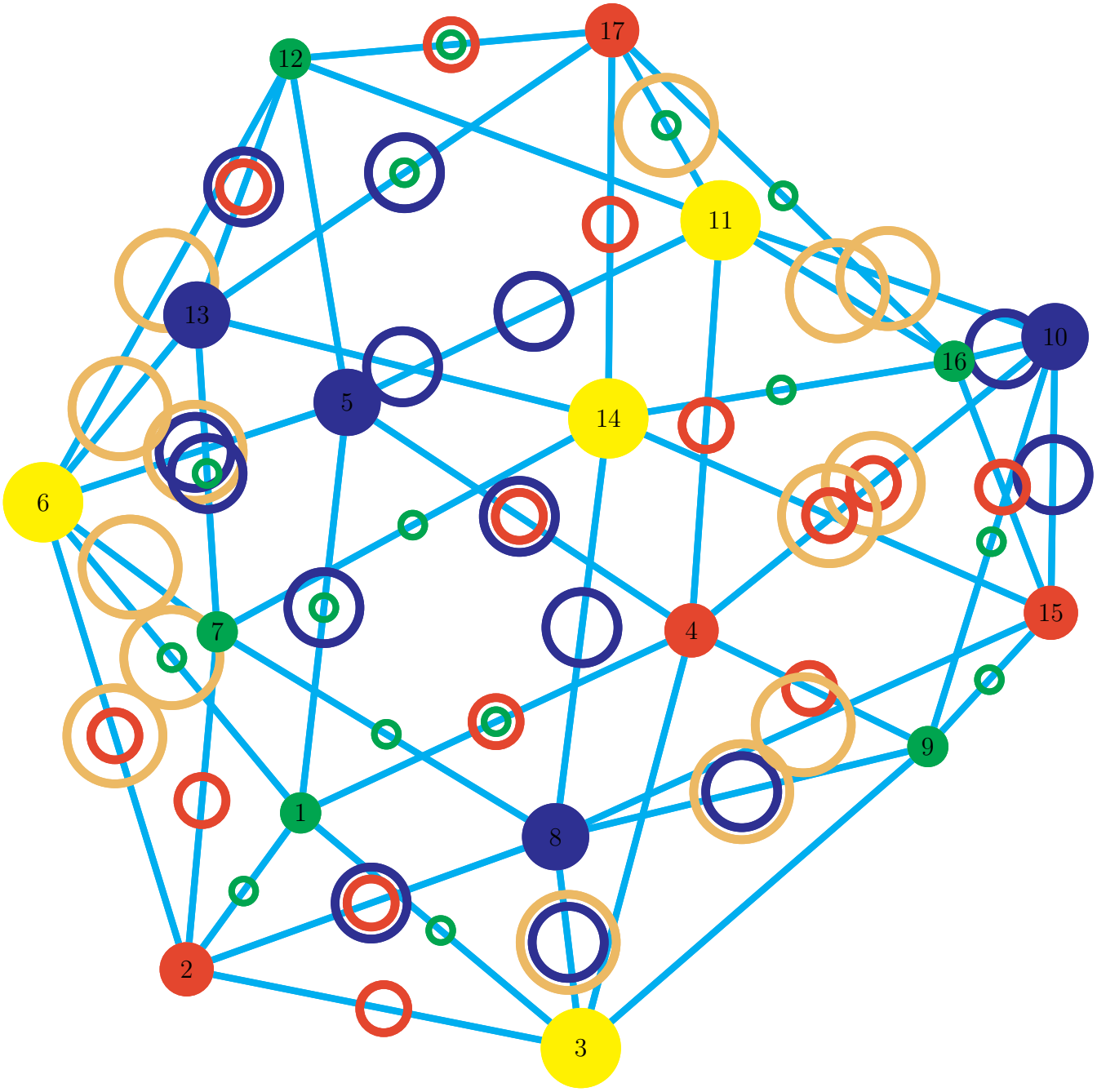

Figure 129: Instructions: 475: color edge 16→14 Green

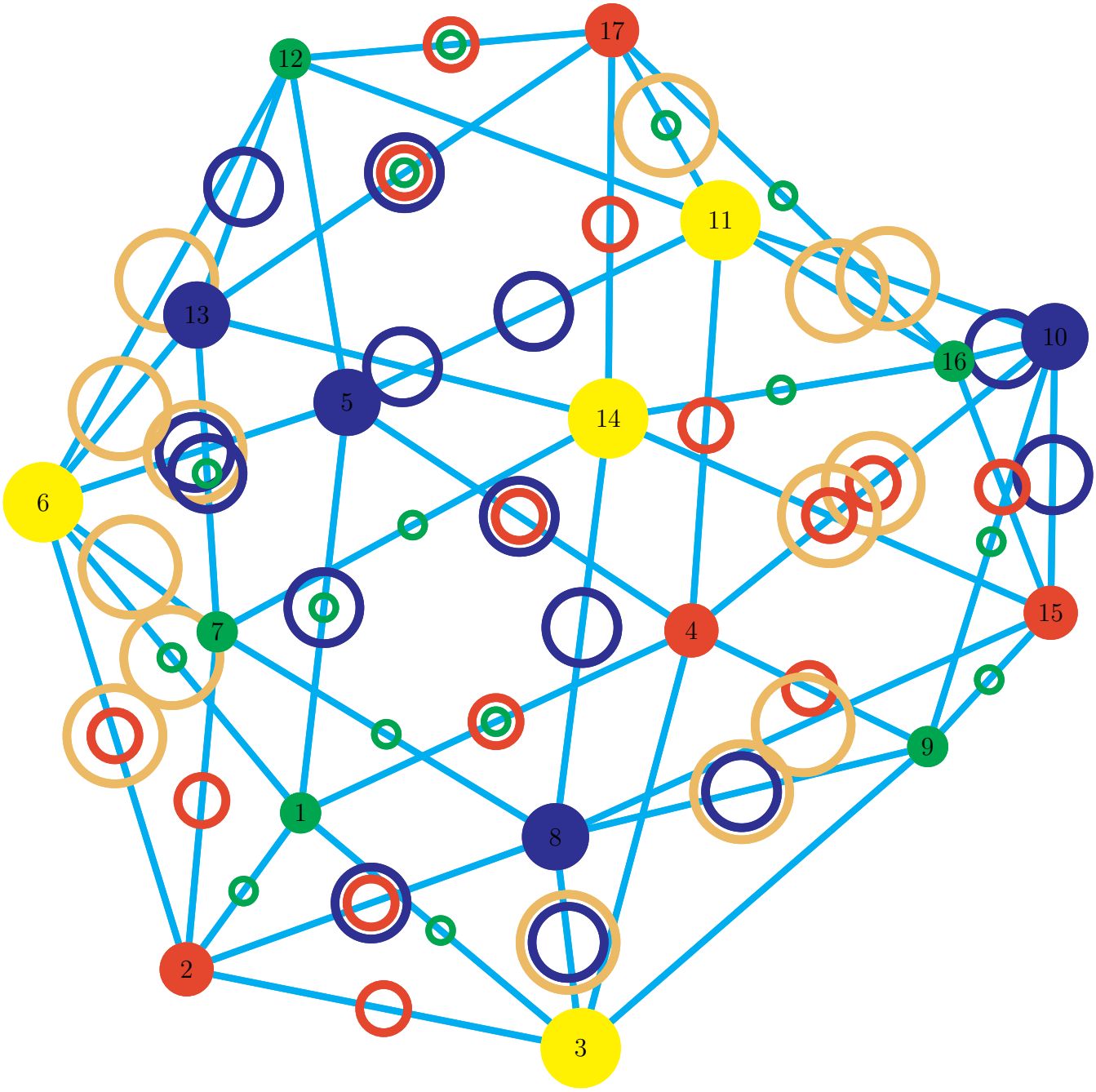

Figure 130: Instructions: 479: color edge 17→13 Red

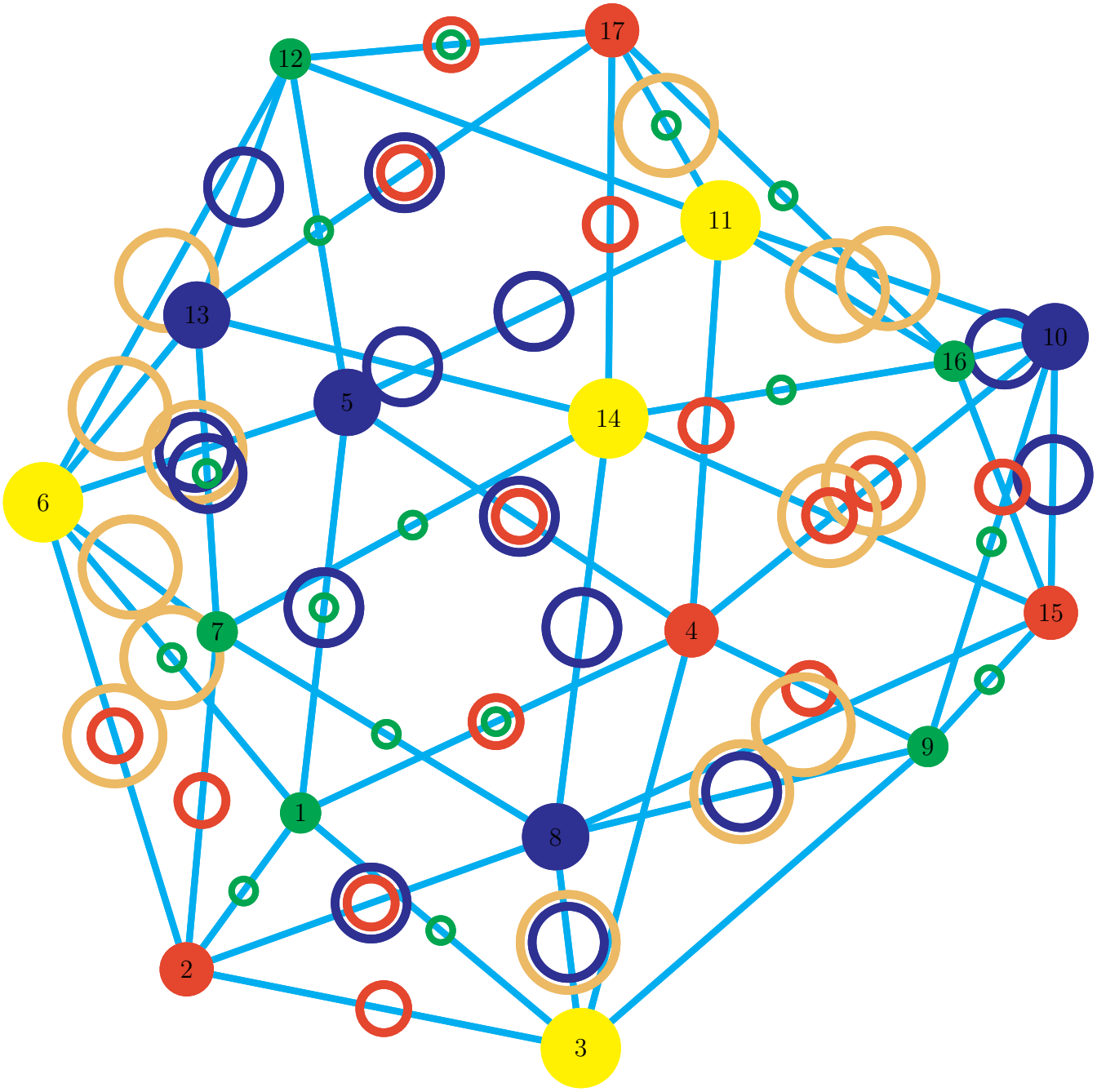

Figure 131: Instructions: 483: color edge 12→5 Green

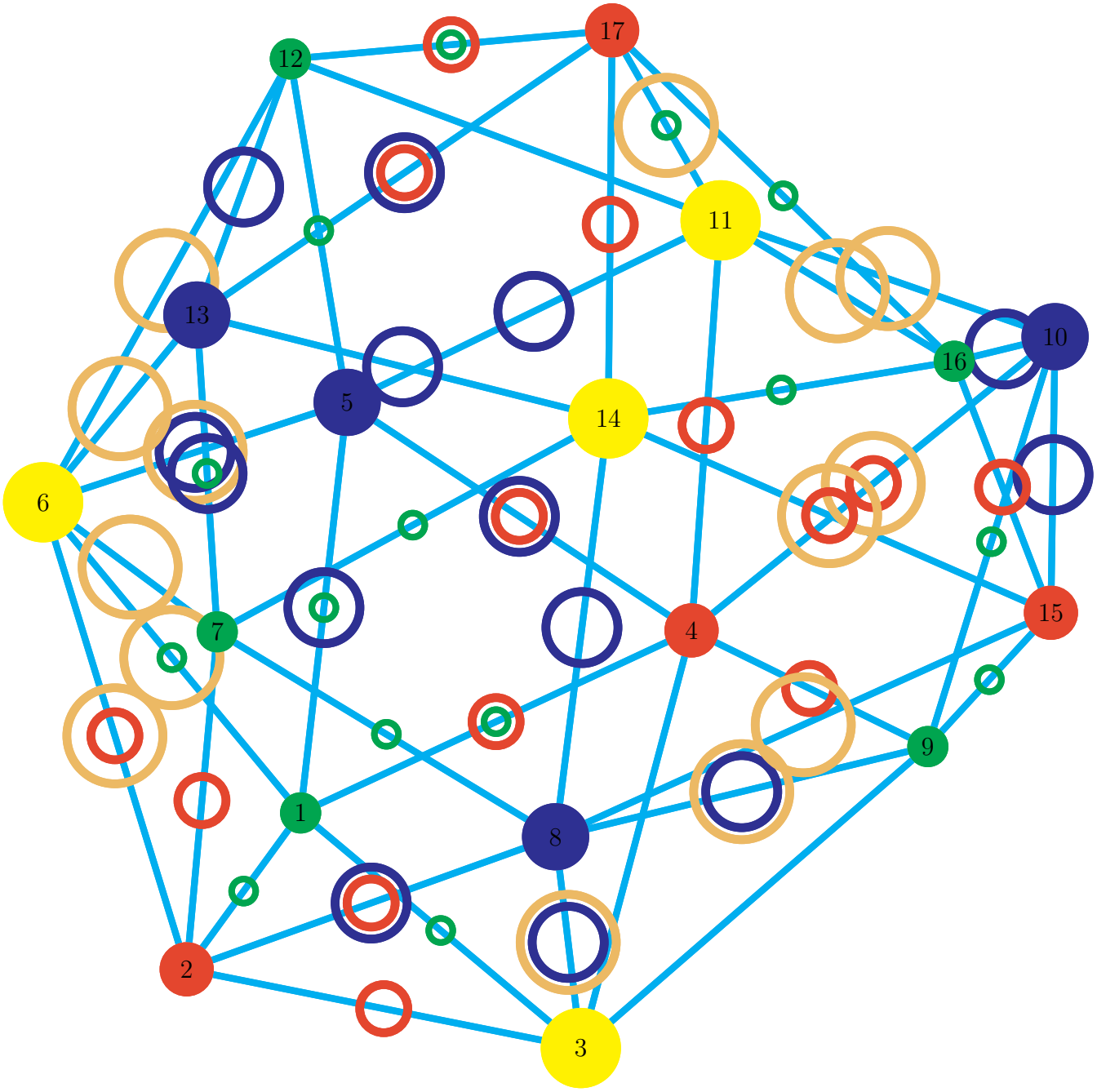

Figure 132: Instructions End:

Finally we have properly four colored the graph